Surface Gravity Squared as Gravitational Boundary Pressure: Acceleration-Length Response, Normal Force Balance, and FRW Screen Work

J. T. Tyler[1, *]

[1] *La Jolla, CA, USA*

On a clock-anchored non-expanding null boundary, the established Einstein normal-rate source admits the branchwise acceleration length $L_\mathrm{s} = c^2/\kappa$. Its Hamilton–Jacobi response is the normal pressure $P_L = \kappa^2/(8\pi G)$, with $F_L = AP_L$ on a uniform cut and $F_L\, dL_\mathrm{s} = P_L\, dV_\perp$. The signed response one-form pulls back to the membrane pressure gradient, while an established Young–Laplace screen equation supplies a genuine normal force balance. The same L_κ carries the classical law $P_L \propto L_\kappa^{-2}$ and the semiclassical law $T \propto L_\kappa^{-1}$, reconstructing the quarter-area entropy density. Rindler, Schwarzschild, Cai–Kim FRW, and a finite dynamical-horizon theorem give complementary realizations. Schwarzschild exhibits an exact thermal–mechanical/free-energy duality; a stationary Wald-entropy extension and new response functions are also given.

a. Canonical result. After a boundary Legendre transform, the established spin-zero Einstein null potential is [1, 2]

$$\Theta_\mu^{(0)} = \frac{1}{8\pi G}\int_{\mathcal{N}} \epsilon_S\, \delta\mu_\xi\, dv, \qquad \mu_\xi = \widehat{\kappa}_\xi + \frac{1}{2}\theta_\xi. \tag{1}$$

The generic source is μ_ξ, not surface gravity alone. Restricting to a prescribed-clock non-expanding sector, $\theta_\xi = \delta\theta_\xi = 0$, and restoring the acceleration-valued $\kappa_\xi = c^2\widehat{\kappa}_\xi$ gives

$$\Theta_\kappa^{(0)} = \frac{c^2}{8\pi G}\int d\tau \int_S \epsilon_S\, \delta\kappa_\xi. \tag{2}$$

On either nonzero branch, $L_\mathrm{s} = c^2/\kappa_\xi$ therefore yields directly

$$\boxed{\Theta_L^{(0)} = -\int d\tau \int_S \epsilon_S\, P_L\, \delta L_\mathrm{s}}, \qquad \boxed{P_L = \frac{\kappa_\xi^2}{8\pi G} = \frac{c^4}{8\pi G L_\mathrm{s}^2}}. \tag{3}$$

For a uniform cut, $F_L = AP_L$. At fixed transverse area, $\delta V_\perp = A\delta L_\mathrm{s}$, so the term is $-P_L\delta V_\perp$: this is the direct ordinary-mechanical reason for the pressure name. Here $L_\kappa = |L_\mathrm{s}| = c^2/|\kappa|$ is used below for positive thermodynamic magnitudes. The area–boost corner pair independently gives $J_\eta\delta\eta = -\Delta\tau F_L\delta L_\mathrm{s}$ [3]. At $\kappa = 0$ the reciprocal chart fails, while $-P_L\delta L_\mathrm{s} = (c^2/8\pi G)\delta\kappa$ remains regular. The novelty is not the chain rule or parent $A\kappa$ potential, but the admissible and mechanically privileged source coordinate, its action response and clock conditions, and the structure it exposes.

b. Surface stress, pullback, and normal balance. Integrating Eq. (3) gives the established boundary potential

$$\begin{aligned} U_\partial &= \frac{Ac^4}{8\pi G L_\mathrm{s}}, \qquad \delta U_\partial = \Pi_\partial\delta A - F_L\delta L_\mathrm{s}, \\ \Pi_\partial &= \frac{c^2\kappa}{8\pi G} = P_L L_\mathrm{s}, \qquad |\Pi_\partial| = P_L L_\kappa. \end{aligned} \tag{4}$$

Π_∂ is the two-dimensional membrane pressure/tension because $-D_A\Pi_\partial$ occurs in the Damour–Navier–Stokes tangential momentum balance [4–6]; P_L is instead the normal force-per-area susceptibility, with $P_L = -\partial\Pi_\partial/\partial L_\mathrm{s} = -\partial|\Pi_\partial|/\partial L_\kappa$ on a fixed-sign branch. For a nonuniform signed field $L_\mathrm{s} : S \to \mathbb{R}\setminus\{0\}$ restricted to one sign branch, view $\Pi(L) = c^4/(8\pi G L)$ on source space. The precise pullback and its magnitude form are

$$\begin{aligned} &L_\mathrm{s}^*(d\Pi) = -P_L dL_\mathrm{s}, \qquad D_A\Pi_\partial = -P_L D_A L_\mathrm{s}, \\ &D_A|\Pi_\partial| = -P_L D_A L_\kappa. \end{aligned} \tag{5}$$

Thus the known tangential pressure gradient is the restriction to the physical cut of the signed source-space response one-form. This organizes the relationship but does not make P_L more general than the parent null source μ.

A complementary normal projection is supplied by the established gravitational-screen Young–Laplace equation [7]. With surface tension $\gamma = -\Pi_\partial = -P_L L_\mathrm{s}$, static balance is

$$\boxed{\Delta p - P_L L_\mathrm{s} H = 0} \tag{6}$$

(up to simultaneous orientation reversal); for $\kappa > 0$ this is $\Delta p - P_L L_\kappa H = 0$. Here H is mean curvature and Δp includes matter and the intrinsic three-curvature term. In the spherical, irrotational, constant-lapse reduction,

$$-\left(D_t + \frac{3}{4}\theta_t\right)\frac{\theta_n}{8\pi G} = \Delta p - P_L L_\mathrm{s} H. \tag{7}$$

Equations (6)–(7) are exact acceleration-length rewritings of an Einstein screen constraint, not a new field equation. They show why no three-dimensional Navier–Stokes equation for P_L is required: Damour governs tangential two-fluid momentum, while Young–Laplace governs normal screen motion.

c. Mechanical–thermal bridge and responses. Choose the positive-κ thermodynamic orientation, so $L_\mathrm{s} = L_\kappa$, $\Pi_\partial = |\Pi_\partial|$, and $U_\partial = |U_\partial| = TS$. Then

$$\begin{aligned} L_\kappa &= \frac{c^2}{|\kappa|} = \frac{\hbar c}{2\pi k_B T}, \\ P_L(L_\kappa) &= \frac{c^4}{8\pi G L_\kappa^2}, \qquad k_B T(L_\kappa) = \frac{\hbar c}{2\pi L_\kappa}. \end{aligned} \tag{8}$$

Thus L_κ is a *mechanical–thermal bridge scale*: the same normalized horizon scale carries a classical gravitational pressure law and a semiclassical quantum-field temperature law. Weighted homogeneity gives

$$U_\partial = TS = \Pi_\partial A = P_L(AL_\kappa) = F_L L_\kappa, \qquad \boxed{\frac{k_B T}{P_L L_\kappa} = 4\ell_\mathrm{P}^2}. \tag{9}$$

Hence $S/A = k_B/(4\ell_\mathrm{P}^2)$: one entropy nat, meaning one unit of S/k_B, occupies $4\ell_\mathrm{P}^2$. The numerical quarter is the ratio of the Einstein $1/(8\pi G)$ boundary normalization to the Hawking–Unruh $1/(2\pi)$ normalization; κ and clock scaling cancel. Wald notes that black-hole entropy need not correspond to state counting "in any usual sense" [8]. Macroscopically, a horizon supplies an observer- or evolution-relative accessible/inaccessible partition; boundary mechanics assigns its energy density and temperature converts that density into entropy. This does not inventory every hidden fact or choose a unique microscopic carrier.

---

* Contact: jttyler101@gmail.com. Public disclosure of the positive FRW pressure/work result (May 10, 2025): `https://x.com/JTT329/status/1921341959009902709`.

At fixed area, $V_\perp = AL_\kappa$ and

$$\left(\frac{\partial V_\perp}{\partial P_L}\right)_A = -\frac{V_\perp}{2P_L}, \qquad \chi_L^{(A)} = \frac{1}{2P_L}, \qquad B_L^{(A)} = 2P_L, \quad (10)$$

so dV/dP is a compliance; its inverse is stiffness. Also $P_L(T) = \pi c^2 k_B^2 T^2/(2G\hbar^2)$ and $dP_L/dT = 2P_L/T$. The source-space Hessian has determinant $-P_L^2 < 0$; this saddle of a boundary ensemble is not by itself a dynamical instability, while the fixed-area modulus is positive.

d. Physical sectors and the energy split. *Rindler:* proper time fixes $\kappa = a$ and $L_\kappa = c^2/a$, the observer–horizon proper distance, so $P_L = a^2/(8\pi G)$ exists in flat spacetime without source mass or curvature. Frodden–Ghosh–Perez use the same local length but hold it fixed [15].

Schwarzschild: Killing time at infinity gives $\kappa = c^2/(2R_H)$, $L_\kappa = 2R_H$, and

$$\boxed{Mc^2 = TS + F_L L_\kappa}, \qquad TS = F_L L_\kappa = \mathcal{F}_H^{\rm on} = \frac{Mc^2}{2}. \quad (11)$$

This is an exact pointwise Euler–Smarr/virial duality, stronger than a relation requiring time averaging. In an ordinary homogeneous-potential system, $2\langle K\rangle = n\langle V\rangle$ and the averaged components are fixed fractions of the total energy; changing that energy need not make one fall while the other rises [12]. Here

$$d(TS) = d(F_L L_\kappa) = \frac{1}{2} d(Mc^2), \quad (12)$$

so adding black-hole mass raises both halves in lockstep. The terms are operationally distinct because $TS = A(\partial U_\partial/\partial A)_L$ and $F_L L = -L(\partial U_\partial/\partial L)_A$ probe different conjugate directions, although the Schwarzschild family locks them equal. It is physically motivated to interpret TS as thermal/kinetic-like and $F_L L_\kappa$ as mechanical/potential–free-energy-like. What remains interpretive is the literal microscopic identification, not the exact co-evolution [13]. The pressure also matches the longitudinal Newtonian field-stress magnitude at the horizon, while Rindler shows that the boundary response is more general than that correspondence.

Cai–Kim FRW: $L_\kappa = R_A$ and [14]

$$\boxed{P_{\rm scr} = \frac{dE_A}{dV_A} = \frac{\rho}{3} = P_L}, \qquad \boxed{dE_A = T_{\rm CK} dS_A - P_{\rm scr} dV_A}. \quad (13)$$

This is positive constrained screen work, not matter pressure. Along the spherical family, $dV_A/dP_{\rm scr} = -3V_A/(2P_{\rm scr})$ and $B_{\rm scr} = 2P_{\rm scr}/3$. The sectoral ratio $U_\partial/E_{\rm sph} = \kappa R/c^2$ explains why Schwarzschild has $U_\partial = E/2$ while Cai–Kim has $U_\partial = E$; correspondingly their physical heat capacities are $-2S$ and $-S$.

Finite dynamics: the APS charge–flux theorem [9] rewritten with $L_{\rm APS} = c^2/\bar\kappa$ gives, in the nonrotating sector,

$$\begin{aligned} E^{(\xi)}_{\rm gws} + E^{(\xi)}_{\rm matt} = \Delta U_\partial = \frac{c^4}{8G}\,\Delta L_{\rm APS}, \\ \Delta(TS) = \Delta(F_L L) = \frac{1}{2}\Delta E_{\rm DH}. \end{aligned} \quad (14)$$

This is a genuine finite constant-force displacement law and exact equal co-evolution of the thermal and mechanical representations. It does not map one boundary leg to matter flux and the other to wave flux.

e. Higher curvature and limits. The stationary $S_W dT$ channel extends directly:

$$P^{(W)}_{L,SdT} = \frac{\hbar c}{2\pi k_B L_\kappa^2}\, s_W, \qquad s_W \equiv S_W/A. \quad (15)$$

For a pure mth-order Lanczos–Lovelock theory, the actual membrane pressure instead obeys $\Pi^{(m)} A/T = [(D-2m)/(D-2)]S_W^{(m)}$, so its scale susceptibility carries the same factor [10, 11]. Einstein gravity is $m = 1$; in critical $D = 2m$ the topological entropy can remain while the local membrane-pressure factor vanishes. Generic dynamical higher-curvature horizons require a separate null-source analysis.

Finally, the isolated signed scale-work one-form has $d_\Gamma(-F_L\delta L_{\rm s}) = -P_L\delta A\wedge\delta L_{\rm s}$; it is therefore not a universal separately stored energy when area and scale vary independently, although the curl is canceled by the area leg in δU_∂. On the opposite orientation, signed stress and work acquire ${\rm sgn}(\kappa)$ while P_L, L_κ, and all magnitudes are unchanged. The core Einstein proposition is exact under its stated clock, cut, sector, action, and counterterm assumptions; other horizon prescriptions support the same algebra only at the level separately established in each sector.

[1] F. Hopfmüller and L. Freidel, Phys. Rev. D **95**, 104006 (2017); **97**, 124029 (2018).

[2] G. Odak, A. Rignon-Bret, and S. Speziale, JHEP **12**, 038 (2023).

[3] S. Carlip and C. Teitelboim, Class. Quantum Grav. **12**, 1699 (1995); E. Bianchi and W. Wieland, arXiv:1205.5325.

[4] T. Damour, in *Proceedings of the Second Marcel Grossmann Meeting* (North-Holland, 1982), p. 587.

[5] R. H. Price and K. S. Thorne, Phys. Rev. D **33**, 915 (1986).

[6] S. Kolekar and T. Padmanabhan, Phys. Rev. D **85**, 024004 (2012).

[7] L. Freidel and Y. Yokokura, Class. Quantum Grav. **32**, 215002 (2015).

[8] R. M. Wald, Gen. Relativ. Gravit. **57**, 87 (2025).

[9] A. Ashtekar, D. E. Paraizo, and J. Shu, Phys. Rev. Lett. **136**, 251405 (2026).

[10] S. Chakraborty and T. Padmanabhan, Phys. Rev. D **90**, 084021 (2014).

[11] S. Kolekar and D. Kothawala, Phys. Rev. D **85**, 064031 (2012).

[12] J. F. Cariñena, F. Falceto, and M. F. Rañada, arXiv:1209.4584.

[13] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).

[14] R.-G. Cai and S. P. Kim, JHEP **02**, 050 (2005); S. A. Hayward, Class. Quantum Grav. **15**, 3147 (1998).

[15] E. Frodden, A. Ghosh, and A. Perez, Phys. Rev. D **87**, 121503(R) (2013).