Horizon boundary mechanics

Acceleration-length work and the equilibrium cosmological screen

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Abstract. Acceleration length expresses the established Einstein boundary response as a normalized force and pressure. Its work term equals thermal cooling and preserves the usual mass first law. De Sitter supplies an equilibrium cosmological reference within the same restricted action; screen pressure and matter flux restate Einstein–FRW dynamics.

1. One potential, two mechanical responses

In four-dimensional Einstein gravity, fix c, G, the physical clock and matching orientation. Let acceleration-valued surface gravity $\kappa > 0$ be uniform on a finite cut of area A. The established boundary potential is

$$L = \frac{c^2}{\kappa}, \qquad U(A, L) = \frac{A\kappa c^2}{8\pi G} = \frac{c^4 A}{8\pi G L}, \qquad M_U = \frac{U}{c^2}. \tag{1}$$

Its differential, before any solution-family restriction, identifies surface stress, length force and normal-scale pressure:

$$\boxed{dU = \Pi\, dA - F_L\, dL,} \qquad \Pi = \left(\frac{\partial U}{\partial A}\right)_L = \frac{U}{A}, \quad F_L = -\left(\frac{\partial U}{\partial L}\right)_A = \frac{U}{L} = M_U\kappa, \tag{2}$$

$$\boxed{P_L = \frac{F_L}{A} = \frac{\Pi}{L} = \frac{\kappa^2}{8\pi G}, \qquad U = \Pi A = F_L L.} \tag{3}$$

For an invertible rate-only coordinate $q(\kappa)$, retain the area source and clock: $-F_q\, dq = (Ac^2/8\pi G)d\kappa$. Requiring $F_q = M_U\kappa$ gives $dq/d\kappa = -c^2/\kappa^2$, hence $q = L + q_0$; $U = F_q q$ fixes $q_0 = 0$, agreeing with Rindler distance. This conditional uniqueness leaves other coordinates valid with transformed forces. The Newtonian field-stress magnitude $g^2/(8\pi G)$ [9] matches P_L at $g = \kappa$; physical identification also requires matching field, observer and normal.

2. Canonical response and cooling are the same scale variation

The established null functional $(8\pi G)^{-1}\int \sqrt{q}(\theta + \kappa)\, d\lambda\, d^2x$ ($c = 1$) reduces to $U\Delta\tau$ for uniform nonexpanding data [10,11]. Choose the rate-source prescription exchanging $-\Pi\delta A$ for $A\delta\Pi$ [1,2]. Fix the physical generator, clock, cuts and duration; impose $\theta = \delta\theta = 0$ and control shear, matter and corner terms. Admitted stationary variations give

$$\boxed{\frac{\Theta_\kappa^{(0)}}{\Delta\tau} = -\Pi\,\delta A + \delta U = A\,\delta\Pi = \frac{Ac^2}{8\pi G}\,\delta\kappa = -F_L\,\delta L.} \tag{4}$$

Here $\Theta_\kappa^{(0)}$ is the selected scalar response, not the full Hamiltonian variation; it does not establish arbitrary independent on-shell source families. The stationary corner check uses $J_\eta = Ac^3/(8\pi G)$ and $\eta = \kappa\Delta\tau/c$, giving $J_\eta\delta\eta/\Delta\tau = -F_L\delta L$ [3].

Where Hawking–Unruh thermality and the Einstein area law apply,

$$T = \frac{\hbar c}{2\pi k_B L}, \quad S = \frac{k_B A}{4\ell_P^2}, \quad \ell_P^2 = \frac{\hbar G}{c^3}, \qquad \boxed{U = TS, \quad \Pi\, dA = T\, dS, \quad -F_L\, dL = S\, dT.} \tag{5}$$

The action contributions $T\,\delta S$ and $S\,\delta T$ are established [1, Eqs. (112)–(113)]. The second has the normalized length-force form: outward scale work is cooling when $dL > 0$. This preserves the mass first law; an extraction or boundary-control protocol requires its own energy accounting.

3. Stationary realizations and radius work

Rindler. $L = c^2/a_{\rm obs}$ is proper horizon distance on the observer's rest slice; planar area is independent.

Schwarzschild. Use infinity-normalized time, ADM mass M, $R = 2GM/c^2$, $A = 4\pi R^2$, $L = 2R$, and $U = Mc^2/2$. For areal volume $V = 4\pi R^3/3$,

$$\boxed{F_L\, dL = F_R\, dR = M\kappa\, dR = P_R\, dV, \qquad F_R = \frac{c^4}{4G}, \quad P_R = 2P_L.} \tag{6}$$

The usual mass law and Smarr relation become

$$d(Mc^2) = T\, dS = 2F_L\, dL, \qquad Mc^2 = TS + F_L L = TS + F_R R = 2U. \tag{7}$$

The equal products represent one potential, not independently stored energies. For asymptotically flat Kerr–Newman, the usual first law and Smarr relation [8] give

$$F_L\, dL = -S\, dT = \tfrac{1}{2}(T\, dS + \Omega_H\, dJ + Q\, d\Phi_H) + J\, d\Omega_H \tag{8}$$

Here Ω_H, J are angular velocity and angular momentum; Φ_H, Q are electric potential and charge. Radius work requires the chosen family. At finite area, $\kappa \to 0$ sends $U, F_L, P_L \to 0$ while $L \to \infty$; the κ response remains the regular description.

4. Einstein–FRW screen and its matter-pressure content

Let ρ, p_m be total bulk energy density and pressure, including vacuum; $H = \dot{a}/a$. For any spatial curvature, define $R_A = c/\sqrt{H^2 + kc^2/a^2}$, $A_A = 4\pi R_A^2$ and areal volume $V_A = 4\pi R_A^3/3$. The equilibrium Cai–Kim assignment is $\kappa_{\rm CK} = c^2/R_A$, $L_{\rm CK} = R_A$ and $T_{\rm CK} = \hbar c/(2\pi k_B R_A)$ [4]. Einstein–FRW gives

$$E_A = E_{\rm MS}(R_A) = \rho V_A = \frac{c^4 R_A}{2G} = U, \qquad F_{\rm FRW} = \frac{c^4}{2G}, \qquad \boxed{P = P_R = P_L = \frac{c^4}{8\pi G R_A^2} = \frac{\rho}{3}.} \tag{9}$$

Along this constrained screen family,

$$F_L\, dL = P\, dV_A = dE_A, \qquad T_{\rm CK}\, dS = 2\, dE_A, \qquad \boxed{dE_A = T_{\rm CK}\, dS - P\, dV_A.} \tag{10}$$

Cai–Kim's crossing heat [4,5], using $\rho = 3P$ and bulk conservation, becomes

$$\boxed{\delta Q_{\rm CK} = A_A(3P + p_m) H R_A\, dt = -3V_A\, dP = T_{\rm CK}\, dS = 2\, dE_A.} \tag{11}$$

The moving sphere instead obeys $dE_A = 3P\, dV_A - \delta Q_{\rm CK}$; crossing flux and total energy change agree with Eq. (10). With $\kappa_E = 8\pi G/c^4$, flat FRW has

$$H^2 = \frac{8\pi G}{c^2} P, \qquad \frac{2\ddot{a}}{ac^2} = -\kappa_E(P + p_m), \qquad \dot{P} + H(\rho + p_m) = 0. \tag{12}$$

Conservation holds for any spatial curvature: $p_m = -3P - \dot{P}/H$ for $H \neq 0$. The evolution of P retains the bulk-pressure information.

5. De Sitter reference and varying vacuum pressure

At an FRW instant, a center-normalized de Sitter reference with $R_0 = R_A(t_0)$ and $\Lambda_0 = 3/R_0^2$ has $L_0 = R_0$ and $P_{L,0} = \rho(t_0)/3$ [6]. At fixed Λ_0, its cosmological action term adds no derivative boundary response, so Sec. 2 applies with consistent orientation. Varying the pure de Sitter radius also varies Λ_0 and requires its bulk variation. The established extended first law [13] gives

$$P_\Lambda = -\frac{\Lambda_0 c^4}{8\pi G} = -3P_{L,0}, \qquad \boxed{V_0\, dP_\Lambda = -3V_0\, dP_{L,0} = T_0\, dS_0.} \tag{13}$$

This compares stationary solutions. At fixed Λ_0, exact de Sitter has $\dot{P} = 0$ and zero crossing flux. The reference does not establish the full moving-FRW canonical response.

6. A finite wall fixes the horizon clock

A Schwarzschild cavity wall at $r_B > R$ prescribes $N_B = \sqrt{1 - R/r_B}$, $A_B = 4\pi r_B^2$ and wall proper time. Then $\kappa_B = \kappa_\infty/N_B$, $L_B = 2RN_B$ and $U_B = T_B S = Mc^2/(2N_B)$. The flat-reference Brown–York law $dE_{\rm BY} = T_B dS - p_{\rm BY} dA_B$ and scaling give [14]

$$U_B = \tfrac{1}{2} E_{\rm BY} + p_{\rm BY} A_B, \qquad F_{L,B}\, dL_B|_A = -\tfrac{1}{2} p_{\rm BY}\, dA_B - A_B\, dp_{\rm BY}. \tag{14}$$

Here $p_{\rm BY}$ is surface pressure. $F_{L,B}$ is conjugate to the wall-normalized horizon length, not the net wall force.

7. Signed Hayward pressure and the earlier FRW equation of state

The dynamical signed rate is $\kappa_H^{\rm s} = -(R_A/2)(\dot{H} + 2H^2 + kc^2/a^2)$ [5,7]. Let $w = p_m/\rho$, $\chi = \kappa_H^{\rm s}/\kappa_{\rm CK} = (3w - 1)/4$, and define the signed radial coefficient

$$P_R^H = \frac{c^2 \kappa_H^{\rm s}}{8\pi G R_A} = \chi P, \qquad \boxed{p_m = P + 4P_R^H,} \qquad P_L^H = \frac{(\kappa_H^{\rm s})^2}{8\pi G} = \frac{(P_R^H)^2}{P}. \tag{15}$$

On a nonzero signed branch, $L_H^{\rm s} = c^2/\kappa_H^{\rm s}$ and $F_L^H = A_A P_L^H$ give $F_L^H\, dL_H^{\rm s} = P_R^H\, dV_A - \frac{3}{4} E_A\, dw$. The earlier FRW equation of state [12], at $\Lambda = 0$ and $\kappa_H^{\rm s} < 0$, is

$$W = \frac{\rho - p_m}{2} = P + \frac{k_B T_H}{2\ell_P^2 R_A} = \boxed{P - 2P_R^H}, \qquad T_H = \frac{\hbar|\kappa_H^{\rm s}|}{2\pi k_B c}. \tag{16}$$

The earlier inverse-square term is P; in de Sitter, $W = 3P$ with vacuum included. The additional identification is $P = F_L/A$ in the equilibrium acceleration-length response.

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