Horizon boundary mechanics

Acceleration-length work and the equilibrium cosmological screen

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Acceleration length gives the established Einstein boundary response a geometrically normalized force–displacement interpretation. Scale work equals thermal cooling. The same stationary action realizes the equilibrium FRW calibration through a de Sitter reference.

Boundary potential and paired mechanical responses

In four-dimensional Einstein gravity, fix c, G, the physical clock and matching orientation; let $\kappa > 0$ be uniform on a finite horizon patch of area A. Define

$$L = \frac{c^2}{\kappa}, \qquad U = \frac{A\kappa c^2}{8\pi G} = \frac{c^4 A}{8\pi G L}, \qquad M_U = \frac{U}{c^2}. \tag{1}$$

Before restricting to a solution family,

$$\boxed{dU = \Pi\, dA - F_L\, dL,} \qquad \Pi = \left(\frac{\partial U}{\partial A}\right)_L = \frac{U}{A}, \quad F_L = -\left(\frac{\partial U}{\partial L}\right)_A = \frac{U}{L} = M_U\kappa, \tag{2}$$

$$P_L = \frac{F_L}{A} = \frac{\Pi}{L} = \frac{\kappa^2}{8\pi G}, \qquad U = \Pi A = F_L L. \tag{3}$$

Among rate-only coordinates $q(\kappa)$ retaining the area source, $F_q = M_U\kappa$ gives $q = L + q_0$; $U = F_q q$ fixes $q_0 = 0$, agreeing with Rindler distance. Other coordinates preserve work with transformed forces.

Selected canonical response and thermal translation

Choose the Einstein boundary prescription exchanging $-\Pi\,\delta A$ for the rate response by the Legendre term $\int U\, d\tau$ in the nonexpanding sector [1]. Fix generator, clock, cuts and interval; impose null expansion $\theta = \delta\theta = 0$, with shear, matter and remaining corner responses fixed or accounted for. Uniform stationary data give

$$\frac{\Theta_\kappa^{(0)}}{\Delta\tau} = -\Pi\,\delta A + \delta U = A\,\delta\Pi = \frac{Ac^2}{8\pi G}\,\delta\kappa = -F_L\,\delta L. \tag{4}$$

Here $\Theta_\kappa^{(0)}$ is the selected scalar action response, not the complete Hamiltonian variation. With $S = k_B A/(4\ell_P^2)$, $\ell_P^2 = \hbar G/c^3$ and $T = \hbar c/(2\pi k_B L)$ [2],

$$\boxed{U = TS, \qquad \Pi\, dA = T\, dS, \qquad -F_L\, dL = S\, dT.} \tag{5}$$

Cooling ($dT < 0$) and outward scale work ($dL > 0$) are the same contribution. This representation preserves the standard mass first law.

Stationary realizations

Rindler: $L = c^2/a_{\rm obs}$ is the proper horizon distance on the observer's rest slice; planar area is independent. *Schwarzschild:* with infinity-normalized time, $R = 2GM/c^2$, $L = 2R$ and $U = Mc^2/2$. For $A = 4\pi R^2$, $dV = A\, dR$,

$$F_L\, dL = F_R\, dR = M\kappa\, dR = P_R\, dV, \qquad P_R = 2P_L, \quad d(Mc^2) = T\, dS. \tag{6}$$

The Smarr relation $Mc^2 = TS + F_L L = TS + F_R R = 2U$ expresses equal products of one potential. *De Sitter:* central proper time gives $L = R_A$ and $U = E_{\rm MS}(R_A)$ on the positive branch.

Einstein–FRW screen, matter flux, and equilibrium reference

For $H = \dot a/a$ and any spatial curvature, $R_A = c/\sqrt{H^2 + kc^2/a^2}$, $A_A = 4\pi R_A^2$ and $V_A = 4\pi R_A^3/3$ (areal volume). Use $\kappa_{\rm CK} = c^2/R_A$, $T_{\rm CK} = \hbar c/(2\pi k_B R_A)$ [3]. The Einstein constraint for $E_A = E_{\rm MS}(R_A)$ gives

$$E_A = U = F_{\rm FRW} R_A = \rho V_A, \quad F_{\rm FRW} = \frac{c^4}{2G}, \qquad \boxed{P = P_R = P_L = \frac{c^4}{8\pi G R_A^2} = \frac{\rho}{3}.} \tag{7}$$

Here ρ, p_m are total bulk energy density and pressure, including vacuum. Along the screen family,

$$F_L\, dL = P\, dV_A = dE_A, \qquad T_{\rm CK}\, dS = 2\, dE_A, \qquad \boxed{dE_A = T_{\rm CK}\, dS - P\, dV_A.} \tag{8}$$

Cai–Kim heat flow [3], using $\rho = 3P$ and conservation, is $\delta Q_{\rm CK} = A_A(3P + p_m)HR_A\, dt = -3V_A\, dP = T_{\rm CK} dS$. The moving sphere obeys $dE_A = 3P\, dV_A - \delta Q_{\rm CK}$; both descriptions agree. For flat FRW, with $\kappa_E = 8\pi G/c^4$, the usual dynamics read

$$H^2 = \frac{8\pi G}{c^2} P, \qquad \frac{2\ddot a}{ac^2} = -\kappa_E(P + p_m), \qquad \dot P + H(\rho + p_m) = 0. \tag{9}$$

Conservation holds for any spatial curvature and gives $p_m = -3P - \dot P/H$ for $H \neq 0$. De Sitter's established action [2] realizes the same response (4), since fixed Λ adds no metric derivatives. Matching $R_0 = R_A(t_0)$ and $\Lambda_0 = 3/R_0^2$ gives the equilibrium reference. Full moving-horizon identification is separate; pure de Sitter radius variations also change Λ_0.

[1] Hopfmüller–Freidel, 1611.03096; Odak–Rignon-Bret–Speziale, 2309.03854.

[2] Parattu–Majhi–Padmanabhan, 1303.1535; Gibbons–Hawking, Phys. Rev. D 15, 2738 and 2752 (1977).

[3] Cai–Kim, hep-th/0501055; Hayward, gr-qc/9710089; Akbar–Cai, hep-th/0609128. Reference metric and derivation: two-page presentation; full companion at jttyler.com.