Horizon boundary mechanics

Area, acceleration length, and horizon work

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Acceleration length $L = c^2/\kappa$ supplies a geometrically normalized work coordinate for the established Einstein boundary response, identifying its conjugate force and normal-scale pressure.

Geometry and the mechanical responses

For a specified clock and positive surface gravity κ, use the horizon area A and

$$L = \frac{c^2}{\kappa}, \qquad U(A, L) = \frac{A\kappa c^2}{8\pi G} = \frac{c^4}{8\pi G}\frac{A}{L}. \tag{1}$$

The specified Einstein boundary prescription fixes this potential [1]. Its complete differential is

$$dU = \frac{c^4}{8\pi GL}\, dA - \frac{Ac^4}{8\pi GL^2}\, dL = \Pi\, dA - F_L\, dL. \tag{2}$$

For the mass equivalent $M_U = U/c^2$, the responses and force density are

$$\Pi = \left(\frac{\partial U}{\partial A}\right)_L = \frac{U}{A}, \qquad F_L = -\left(\frac{\partial U}{\partial L}\right)_A = \frac{U}{L} = M_U\kappa, \tag{3}$$

$$P_L = \frac{F_L}{A} = \frac{\Pi}{L} = \frac{\kappa^2}{8\pi G}, \qquad U = \Pi A = F_L L = P_L AL. \tag{4}$$

Surface stress pairs with area change; force pairs with horizon-scale change: $A\, d\Pi = -F_L\, dL$. Rindler fixes the length normalization; other coordinates preserve work with transformed coefficients.

Thermal translation and stationary horizons

The Hawking–Unruh temperature and Einstein area entropy give

$$T = \frac{\hbar c}{2\pi k_B L}, \qquad S = \frac{k_B A}{4\ell_P^2}, \quad \ell_P^2 = \frac{\hbar G}{c^3}, \qquad U = TS, \quad \Pi\, dA = T\, dS. \tag{5}$$

Rindler: $L = c^2/a_{\text{obs}}$ is the normal horizon distance on the instantaneous rest slice; planar patch area A is independent. *Schwarzschild:* $A = 4\pi R^2$, $L = 2R$, $U = Mc^2/2$, with time normalized at infinity. For areal volume $V = 4\pi R^3/3$,

$$F_L\, dL = F_R\, dR = M\kappa\, dR = P_R\, dV, \qquad P_R = 2P_L = \frac{\kappa^2}{4\pi G}. \tag{6}$$

Here $dL/L = dR/R$ is the same fractional horizon-size change; $U = 3P_R V$. The decomposition $Mc^2 = TS + F_L L$ has both products equal to U and retains $d(Mc^2) = T\, dS$.

Cosmological payoff

For the instantaneous equilibrium FRW screen, $L = R_A$, so $P \equiv P_R = P_L$. With $V = 4\pi R_A^3/3$, the Einstein–Misner–Sharp identification gives [2]

$$U = E = \frac{c^4 R_A}{2G} = \rho V, \qquad P = \frac{\rho}{3}, \qquad \boxed{dE = T\, dS - P\, dV.} \tag{7}$$

For flat FRW, $R_A = c/H$ and the expansion, acceleration, and conservation laws are

$$\boxed{H^2 = \frac{8\pi G}{c^2}P}, \qquad \boxed{\frac{2\ddot{a}}{ac^2} = -\kappa_E(P + p_m)}, \qquad \boxed{\dot{P} + H(\rho + p_m) = 0}, \tag{8}$$

Here $H = \dot{a}/a$, $\kappa_E = 8\pi G/c^4$, and ρ, p_m are total bulk energy density and pressure, including vacuum; p_m is distinct from P. For curved FRW, $R_A = c/\sqrt{H^2 + kc^2/a^2}$ and $1/R_A^2 = \kappa_E P$.

Canonical basis and scope

For the selected mixed action, fixed clock and cuts, tangent nonexpansion ($\theta = \delta\theta = 0$), and controlled other responses, uniform stationary data give [1]

$$\frac{\Theta_\kappa^{(0)}}{\Delta\tau} = \frac{Ac^2}{8\pi G}\,\delta\kappa = -\frac{Ac^4}{8\pi GL^2}\,\delta L = -F_L\,\delta L. \tag{9}$$

Here $\Theta_\kappa^{(0)}$ is the selected scalar boundary contribution to the action variation. Rewriting this established response using $L = c^2/\kappa$ identifies F_L as the conjugate force for admitted source variations; U need not equal the entire Hamiltonian. Stationary Rindler, Schwarzschild and de Sitter realize this sector. Evolving FRW uses a separate instantaneous screen assignment. Thermal input is unnecessary for F_L and P_L.

[1] Hopfmüller–Freidel, 1611.03096; Odak et al., 2309.03854; Parattu et al., 1303.1535.

[2] Cai–Kim, hep-th/0501055; Akbar–Cai, hep-th/0609128. Details and scope: the companion seven-page and full manuscripts.