Horizon mechanics: an explanatory guide

Boundary actions, canonical responses, acceleration length, and horizon scope

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Contents

1 The organizing picture **2**
- 1.1 The contribution and the choice of work coordinate . . . 3
- 1.2 Acceleration length as a physical scale . . . 3
- 1.3 The same work in the horizon radius . . . 4

2 From the action to a response **5**
- 2.1 Action, equations, and boundary terms . . . 5
- 2.2 Symplectic potential, not potential energy . . . 5
- 2.3 What is a charge, and when is it a Hamiltonian? . . . 5

3 The null canonical derivation, term by term **6**
- 3.1 Null hypersurface, cut, and generator . . . 6
- 3.2 Inaffinity and surface gravity . . . 6
- 3.3 Expansion, shear, and the scalar combination . . . 7
- 3.4 Area versus rate polarization . . . 7
- 3.5 The explicit local action prescription . . . 8
- 3.6 The force as the normalized length response . . . 8
- 3.7 What makes a source variation permitted? . . . 9
- 3.8 Hamilton–Jacobi meaning and units . . . 9

4 The stationary corner interpretation **10**
- 4.1 Boost angle and its momentum . . . 10
- 4.2 The length momentum explicitly . . . 10
- 4.3 Why $U_\partial L_s = cJ_\eta$ is revealing . . . 10
- 4.4 Only afterward: the thermal dictionary . . . 11

5 Which horizons are actually covered? **11**
- 5.1 A marginal cut is not a nonexpanding null worldtube . . . 12
- 5.2 What already holds in FRW . . . 12

6 Can the canonical argument be extended? **12**
- 6.1 What can already be said on an expanding null boundary . . . 12
- 6.2 What a full moving-boundary theorem would have to establish . . . 13

7 Why ζ_B is defined that way **13**

8 How the pressure enters membrane mechanics **14**

9 The selected clock and boundary energy **15**

10 Why Hayward work is not CK work **15**
10.1 Signed radial pressure and source reconstruction 16
10.2 Causal history in the two pressure variables 17
10.3 Rates, clocks and levels of support 17
10.4 Precise antecedents and the canonical source trail 17

11 Reading the full paper: results and appendices **18**

12 What the revised presentations do **19**

13 Source map and reading links **19**

1 The organizing picture

The papers concern **one gravitational theory, Einstein gravity, examined through specified boundary descriptions**. Area describes how much horizon surface is counted. Acceleration length $L_s = c^2/\kappa$ expresses the same normalized surface-gravity scale as a length. The selected Einstein boundary potential is

$$U_\partial = \frac{A\kappa c^2}{8\pi G} = \frac{Ac^4}{8\pi G L_s}.$$

The contribution is the action-supported interpretation of acceleration length as a geometrically normalized work coordinate, with a conjugate boundary force and normal-scale pressure. The complete differential pairs surface stress with area change and length force with horizon-scale change:

$$dU_\partial = \frac{U_\partial}{A}\,dA - \frac{U_\partial}{L_s}\,dL_s = \Pi_\partial\,dA - F_L\,dL_s.$$

Surface stress is the area coefficient. The length-conjugate force is the negative length derivative at fixed area, and its force per area is the normal-scale pressure:

$$\Pi_\partial = \frac{U_\partial}{A}, \qquad F_L = \frac{U_\partial}{L_s} = M_U\kappa, \qquad M_U = \frac{U_\partial}{c^2},$$

$$P_L = \frac{F_L}{A} = \frac{\Pi_\partial}{L_s} = \frac{c^4}{8\pi G L_s^2} = \frac{\kappa^2}{8\pi G}.$$

The developed force–pressure interpretation uses an established gravitational response. The area and length responses are not separate assumptions, and pressure is the density of the length force, not a third independent canonical pair. The identities $U_\partial = \Pi_\partial A = F_L L_s = P_L A L_s$ and $d\Pi_\partial = -P_L\,dL_s$ collect the same mechanics. For spherical volume $dV = A\,dR$, transform the work coefficient with the length: $F_R = F_L\,dL_s/dR$ and $P_R = F_R/A$. Equilibrium FRW has $P_R = P_L$; Schwarzschild has $P_R = 2P_L$, so both admit exact radial and volume work. The product AL_s remains distinct from the ordinary spherical volume.

Canonical force means an action response. With the action, physical clock and allowed variations specified, F_L is the coefficient of the normalized length-source variation per unit boundary time:

$$\delta I_{\rm os}\big|_{\text{other responses controlled}} = -\int d\tau\, F_L\,\delta L_s.$$

This gives the force its canonical role without requiring U_∂ to equal the entire Hamiltonian. The potential supplies the same fixed-area derivative; the action identifies which controlled variation that derivative describes. Uniform cut data are understood in this integrated force form. Explicit local prescription.

The primary canonical justification is the specified **nonexpanding Einstein null-boundary source response**. A change of rate becomes the same length-work coefficient; the area measure identifies its pressure density. Stationary Rindler, Schwarzschild and consistently normalized de Sitter realize that sector. The stationary area–boost interpretation uses the same clock and length: $J_\eta = Ac^3/(8\pi G)$ and $U_\partial = J_\eta/(L_s/c)$. The action coefficient contains c^3; its inverse timescale supplies the additional c in the energy coefficient.

Equilibrium FRW retains its separate Einstein–FRW/Misner–Sharp identification, with $L = R_A$, $U = E_{\mathrm{MS}}$ and $P = \rho/3$. Its exact changing-screen identities do not assert a complete moving-boundary canonical theorem. Surface stress, normal-scale pressure and bulk matter pressure remain distinct.

The chapters below supply the technical definitions and scope behind this physical sequence. Unchanged detailed source locators refer to their original archived editions unless explicitly marked with a revision date. Source map.

1.1 The contribution and the choice of work coordinate

The established area–surface-gravity differential is the input. The force–length representation identifies its normalized displacement, ties the response to a specified gravitational action, and relates it to radius and volume work in the stated realizations. The comparison with earlier work concerns this combined interpretation and operation: the selected source, clock, allowed variation, and displacement conversion. The earlier rate response, local inverse-distance energy, boost construction, and published special-case pressure coefficients are acknowledged individually.

Why acceleration length? Among lengths built only from c and the prescribed acceleration-valued κ, without another scale, dimensional analysis gives $q = Cc^2/\kappa$. Rindler distance fixes $C = 1$. A different coordinate is still valid when its response transforms: $F_q\,dq = F_L\,dL$. It changes the direct geometric interpretation, not the underlying physics. The action establishes the response; this normalized length selects its mechanical presentation.

Which claim has which support? The potential determines the full state differential. The selected nonexpanding-null prescription identifies the action-source response. The equilibrium FRW realization uses the Einstein–Misner–Sharp relation and its separately specified rate. These results are compatible without asserting one unrestricted moving-boundary theorem.

How does it compare with earlier macroscopic work? Padmanabhan's radial work uses matter pressure in a horizon field-equation identity. The force here is conjugate to a scale variation of the boundary potential; it is nonzero even in vacuum Schwarzschild. Frodden–Ghosh–Perez already give local inverse-distance energy at fixed observer distance; the present construction admits the prescribed scale variation while retaining the appropriate clock.

What is gained without a new prediction? A compact common state law identifies responses, distinguishes energies, and shows when radius work is sufficient. That explanatory content can be assessed within established gravity. The identities $F_L = U/L = M_U\kappa$ are not independent tests; novel empirical claims would require additional dynamics. Exact consequences, conditional extensions, and interpretive suggestions are kept separate.

Read Questions and objections for the detailed answers, including boundary-prescription dependence, the Hayward rate, the stationary Wald-entropy extension, and the evidence needed for priority claims.

1.2 Acceleration length as a physical scale

Surface gravity supplies the normalized rate; acceleration length expresses its reciprocal mechanical scale. For the magnitude boost rate $\Omega_\kappa = |\kappa|/c$, define

$$t_\kappa = \frac{1}{\Omega_\kappa} = \frac{c}{|\kappa|}, \qquad L_\kappa = ct_\kappa = \frac{c^2}{|\kappa|}.$$

For a uniformly accelerated Rindler observer, $\kappa = a$ and rapidity changes by one in proper time $t_\kappa = c/a$. The corresponding light-travel scale is $L_\kappa = c^2/a$. It is also the perpendicular distance to the planar horizon on the observer's instantaneous rest slice. Rapidity is a boost angle, so this does not mean reaching the speed of light. The transverse patch area is independently chosen; L_κ is not a spherical radius.

Area and acceleration scale therefore carry different information. At fixed area, increasing L_κ lowers the magnitude surface gravity and boundary energy per area. The differential records that response as length work. A spherical family can relate the two geometric scales, and then its radius gives the most direct description of changing horizon size. The general reciprocal chart applies on $\kappa \neq 0$; the positive spherical examples below use $L = L_s = L_\kappa$. Stationary clock and boost discussion.

1.3 The same work in the horizon radius

For Schwarzschild with time normalized at infinity,

$$R_H = \frac{2GM}{c^2}, \qquad \kappa = \frac{c^2}{2R_H}, \qquad L = 2R_H, \qquad U_\partial = \frac{Mc^2}{2}.$$

The corresponding length and radial coefficients are

$$F_L = \frac{U_\partial}{L} = \frac{M\kappa}{2}, \qquad F_R = 2F_L = \frac{U_\partial}{R_H} = M\kappa,$$

$$P_R = \frac{F_R}{A} = 2P_L = \frac{\kappa^2}{4\pi G}.$$

Since $A = 4\pi R_H^2$ and $dV = A\,dR_H$ for $V = 4\pi R_H^3/3$, the same work and complete differential become

$$\boxed{F_L\,dL = M\kappa\,dR_H = P_R\,dV,}$$

$$\boxed{dU_\partial = \Pi_\partial\,dA - M\kappa\,dR_H = \Pi_\partial\,dA - P_R\,dV.}$$

Both terms now describe changes of the same horizon sphere. Its areal radius measures geometric size, rather than a universal proper distance from an observer. The factor of two in the pressure converts the displacement coefficient; it does not double a material stress. The especially simple connection is

$$\frac{dL}{L} = \frac{dR_H}{R_H}, \qquad F_L\,dL = U_\partial \frac{dL}{L} = U_\partial \frac{dR_H}{R_H}.$$

No extra surface moves at $2R_H$; both expressions describe the same fractional horizon-size change. This radial form retains the selected canonical response: $Ac^2\,\delta\kappa/(8\pi G) = -F_R\,\delta R_H = -P_R\,\delta V$ under the same stated source and variation conditions.

The subtraction belongs to dU_∂, with $U_\partial = Mc^2/2$. The ordinary mass law remains $d(Mc^2) = T\,dS$. Also, A and V depend on the same radius along this physical family, so the two terms do not introduce independently variable area and volume.

In equilibrium FRW, $L = R_A$, so the length and radial coefficients already coincide: $F_R = F_L = M\kappa$ and $P_R = P_L = \rho/3$, using the enclosed Misner–Sharp mass and energy density. Its work is $F_L\,dL = F_R\,dR_A = P_R\,dV_A$. In both fixed-ratio spherical families,

$$U_\partial = F_L L = F_R R = \Pi_\partial A = 3P_R V.$$

If $\alpha_R = L_s/R$ varies instead, retain $F_L\,dL_s = (U_\partial/R)\,dR + U_\partial\,d\ln|\alpha_R|$. Thus L states the common mechanics, while R makes the specified spherical realization explicit. Force · Pressure · Fractional change.

2 From the action to a response

2.1 Action, equations, and boundary terms

An action assigns a number with units of action (energy times time) to a history of fields and a spacetime region. Schematically,

$$I[g,\phi] = I_{\mathrm{EH}}[g] + I_{\mathrm{matter}}[g,\phi] + I_{\mathrm{boundary}} + I_{\mathrm{corner}}.$$

The Einstein–Hilbert part contains spacetime curvature. Matter terms describe the material fields. Boundary and corner terms make the variation appropriate to the data specified on the region's boundary. A corner is a two-dimensional intersection of boundary hypersurfaces, not necessarily a sharp edge in ordinary three-dimensional space.

Varying an action has the schematic structure

$$\delta I = \int_M E \cdot \delta\Phi + \int_{\partial M} \theta(\Phi, \delta\Phi) + \delta I_{\mathrm{boundary/corner}}.$$

Φ collectively denotes the fields; $E = 0$ denotes their equations of motion. The boundary expression depends on variations of the fields and their geometry. This is why the action supplies both bulk equations and boundary response data. A coefficient in the boundary variation does not replace all the bulk equations.

On shell means that the fields satisfy the equations. It does not mean that the action, every boundary contribution, or every variation of the boundary data vanishes. The action is stationary under the variations appropriate to a fixed boundary-value problem. Comparing solutions with different prescribed boundary sources is a different operation.

2.2 Symplectic potential, not potential energy

The integrated boundary variation Θ is called a symplectic potential. It is a **one-form on field space**: give it a small change of the fields and it returns a linear response. Its name should not be confused with the energy-valued potential U_∂.

The ordinary-mechanics prototype is $p\,\delta q$. The coefficient p is conjugate to the varied coordinate q. In field theory there can be one pair at every boundary point, and an integration over the boundary replaces a finite sum. The field-space exterior derivative

$$\Omega = \delta\Theta$$

encodes the canonical pairings. It is a two-form: it compares two independent variations antisymmetrically. Gauge directions can make it degenerate; before those directions are removed, "presymplectic" is the precise term. This Ω is not the boost rate Ω_κ.

The Einstein null-boundary potential has scalar, vector, and trace-free tensor sectors. These are geometric classifications under transformations within a cut. They are not three newly postulated species of matter and do not imply three unconstrained bulk graviton polarizations. [HF17; HF18]

2.3 What is a charge, and when is it a Hamiltonian?

A charge is the quantity associated with a specified transformation. Electric charge generates an internal gauge transformation; an angular momentum generates rotation; a gravitational boost charge generates a change of the normal frame. Its units depend on the parameter being generated. A dimensionless angle naturally has an action-valued conjugate. A time parameter naturally has an energy-valued generator.

In covariant gravity, the Noether current associated with a vector field ξ has the schematic form $\mathcal{J}_\xi = \theta(g, \mathcal{L}_\xi g) - \iota_\xi \mathcal{L}$. On shell it can be written locally as the exterior derivative of a surface-charge form Q_ξ, subject to the usual constraints and conventions. Integrating that form over a cut gives a charge contribution.

A **Hamiltonian** is a generator whose variation is determined by contraction of the symplectic form with the chosen evolution. It exists as a state function only when that variation is integrable with the specified boundaries and flux treatment. A typical adjusted surface expression is

$$\delta H_\xi = \int_S (\delta Q_\xi - \iota_\xi \theta - Q_{\delta\xi}),$$

with additional transport or flux terms when required. The last term matters when the generator itself depends on the fields. Therefore the Noether charge, the complete Hamiltonian, and U_∂ are not synonyms. A boundary potential can be a definite contribution without being the total energy of the spacetime. [IW94; original full paper §III.B]

A coefficient of a length variation can therefore have a canonical role even when the associated energy-valued boundary potential is only one contribution to the system's energy accounting. For the selected action, the force is $-\delta I_{\rm os}/\delta L_s(\tau)$ with the time measure understood. Establishing that response and establishing a complete Hamiltonian generator are distinct tasks. In Schwarzschild, the potential is $Mc^2/2$ while the ADM energy is Mc^2; both statements remain consistent with the selected length response.

The same point distinguishes ADM mass at infinity, Brown–York energy conjugate to a boundary clock, and Misner–Sharp energy in spherical symmetry. They answer related but different geometric questions; identical units are not enough to identify them.

3 The null canonical derivation, term by term

3.1 Null hypersurface, cut, and generator

A hypersurface is a three-dimensional subset of four-dimensional spacetime. A null hypersurface has a null normal that is also tangent to it. Its generators are lightlike curves lying within it. A spacelike cut S is a two-dimensional cross-section. The horizon's area refers to such a cut, not to the three-dimensional worldtube swept out by its history.

Let q_{AB} be the positive two-metric on a cut, q its determinant, and

$$\epsilon_S = \sqrt{q}\, d^2x, \qquad A = \int_S \epsilon_S.$$

The capital indices here label the two cut coordinates; the index A is not the total area. A selected generator ξ^a tells us how cuts are followed. Its normalization fixes the parameter along the null generators and must be distinguished from an arbitrary auxiliary null-frame convention.

3.2 Inaffinity and surface gravity

With a generator parameter measured in length units, define

$$\nabla_\xi \xi^a = \hat{\kappa}\, \xi^a, \qquad \kappa = c^2 \hat{\kappa}.$$

The operator $\nabla_\xi = \xi^b \nabla_b$ differentiates along the generator. If its parameter is affine, the left side is zero. A nonzero $\hat{\kappa}$ measures failure of that chosen parameter to be affine: the derivative points along the same null direction rather than perpendicular to it. A null generator has no proper time and no ordinary four-acceleration of a massive observer. Surface gravity is not automatically a local accelerometer reading.

$\hat{\kappa}$ has units of inverse length; κ has units of acceleration. On a stationary Killing horizon, the prescribed Killing normalization supplies the usual surface gravity. For Rindler, observer proper time fixes the normalization so $\kappa = a$. For a Schwarzschild horizon, the standard clock is normalized at infinity. A generic null generator can be reparameterized, which changes its inaffinity; the physical clock prescription is therefore essential.

3.3 Expansion, shear, and the scalar combination

The expansion is the fractional change of area along the null generators:

$$\theta = \frac{1}{\sqrt{q}}\mathcal{L}_\xi\sqrt{q}, \qquad \mathcal{L}_\xi\epsilon_S = \theta\epsilon_S.$$

A small bundle of generators with $\theta > 0$ spreads out; $\theta < 0$ contracts. Shear describes distortion of the cut's shape at fixed infinitesimal area. The scalar sector retains area change and normal parameterization, whereas the trace-free tensor sector retains shape change. With the same length parameter, θ also has units of inverse length.

In four-dimensional Einstein gravity the scalar combination is

$$\mu = \hat{\kappa} + \frac{1}{2}\theta.$$

The one-half is the four-dimensional specialization of $(D-3)/(D-2)$ in the cited decomposition. It comes from the geometric split of the null symplectic potential, not a choice made to obtain P_L. The term "mixed" refers to the combination of intrinsic area data and derivative/extrinsic rate data entering the boundary prescription. It does not mean "mixed quantum state." [HF17; HF18; ORS23]

3.4 Area versus rate polarization

In this subsection set $c = 1$. The scalar area polarization is

$$\Theta_A^{(0)} = -\frac{1}{8\pi G}\int_{\mathcal{N}} \mu\,\delta\epsilon_S\,dv.$$

The superscript (0) means scalar or spin zero, not perturbative order. The parameter v runs along the generators. This expression treats area as the displayed configuration variable and the negative rate as its conjugate response.

Choose the boundary Legendre term

$$B_\mu = \frac{1}{8\pi G}\int_{\mathcal{N}} \mu\epsilon_S\,dv.$$

Its exact variation changes the potential to

$$\Theta_\mu^{(0)} = \Theta_A^{(0)} + \delta B_\mu = \frac{1}{8\pi G}\int_{\mathcal{N}} \epsilon_S\,\delta\mu\,dv.$$

The elementary identity behind this is $-p\,\delta q + \delta(pq) = q\,\delta p$. Area and rate have exchanged roles as the displayed source. Because $\delta^2 B_\mu = 0$, the symplectic two-form is unchanged. This is a genuine **change of polarization**. The subsequent substitution $L_s = c^2/\kappa$ is an invertible source-coordinate change inside the chosen polarization, not another Legendre transform.

These are not two new scalar degrees of freedom. They are different descriptions of the same scalar pairing. Boundary conventions still matter: an additional source-dependent action term can shift a response, and corner terms must be treated consistently. [Original full paper §II.A–B; ORS23]

3.5 The explicit local action prescription

The compact paper states the selected response, while Appendix I of the full manuscript gives its local metric calculation. The prescription can be identified without assuming that the energy function alone chooses an action. In geometric units $c = 1$, use a fixed Gaussian-null chart

$$ds^2 = -2r\hat{\kappa}\,du^2 + 2\,du\,dr + 2r\beta_A\,du\,dx^A + q_{AB}\,dx^A dx^B,$$

with null boundary $r = 0$, generator ∂_u, and boundary orientation $-dr$. The cut metric is q_{AB}; $\epsilon_S = \sqrt{q}\,d^2x$; $\theta = \partial_u \log\sqrt{q}$; and $\mu = \hat{\kappa} + \theta/2$. The local Einstein–Hilbert current fixes the scalar subtraction and rate polarization:

$$B_0 = \frac{1}{8\pi G}\int_{\mathcal{N}} \epsilon_S(\hat{\kappa} + \theta)\,du, \qquad B_\mu = \frac{1}{8\pi G}\int_{\mathcal{N}} \epsilon_S \mu\,du,$$

$$I_{\text{rate}} = I_{\text{EH}} - B_0 + B_\mu.$$

Their difference is the endpoint term $-(A_f - A_i)/(16\pi G)$. Its variation cancels for variations tangent to nonexpansion throughout the fixed interval; it does not require that each endpoint area be separately fixed. The scalar rate response is $\int \epsilon_S\,\delta\mu\,du/(8\pi G)$, with shear and the other boundary responses retained or controlled.

Keep the null location, generator/normal convention, chart, identified cuts and integration domain fixed. Impose $\theta = \delta\theta = 0$. For isolating the fixed-area rate response, fixing the cut metric variation and its generator derivative is a sufficient stronger local subcase; matter, other boundaries and corners must also be controlled. Restoring units uses $du = c\,d\tau$ and $\hat{\kappa} = \kappa/c^2$. The physical time label τ is not proper time along a null generator.

An on-shell interpretation compares admitted solutions with these boundary data. This local calculation does not prove that every arbitrarily prescribed source function has such a solution. The complete current, endpoint identity, and counterexamples to dropping the restrictions are given in the full manuscript, Appendix I.

3.6 The force as the normalized length response

Restrict to a nonexpanding null sector and to variations remaining in it:

$$\theta = 0, \qquad \delta\theta = 0.$$

Then $\mu = \hat{\kappa}$. With the prescribed physical time label τ, the scalar response becomes

$$\Theta^{(0)}_\kappa = \frac{c^2}{8\pi G}\int d\tau \int_S \epsilon_S\,\delta\kappa.$$

For uniform stationary data and a fixed duration,

$$\frac{\Theta^{(0)}_\kappa}{\Delta\tau} = \frac{Ac^2}{8\pi G}\delta\kappa.$$

This is the normal-rate term in the full differential of the displayed boundary potential. Since

$$\delta\kappa = -\frac{c^2}{L_s^2}\delta L_s,$$

it follows directly that

$$\Theta_L^{(0)} = -\int d\tau \int_S \epsilon_S P_L \delta L_s, \qquad P_L = \frac{c^4}{8\pi G L_s^2} = \frac{\kappa^2}{8\pi G}.$$

For a uniform cut, $F_L = AP_L$ and $\Theta_L^{(0)} = -\int F_L \delta L_s \, d\tau$.

There is no second algebraic hurdle. A correctly identified canonical one-form transforms by the chain rule, and its two-form transforms consistently. What is not supplied by the chain rule is the choice of physical boundary, clock, polarization, or allowed source space. Differentiating an arbitrary energy-valued function would not establish those ingredients. The canonical derivation establishes the match for the stated ensemble; it does not certify every other horizon prescription by the same notation.

The complete dU_∂ is exact; the selected rate leg is not generally exact when A and L_s vary independently. Identifying one leg of dU_∂ with the source response is not identifying the entire differential with the symplectic potential. [Original full paper §§II.B and VII.A]

3.7 What makes a source variation permitted?

"Permitted" means compatible with the specified variational problem, not independently assumed to be a physical displacement of matter. In this construction the boundary remains null; the variations remain tangent to the chosen nonexpanding sector; cuts and generator normalization are identified by the prescribed clock/reference construction; matter and the other boundary responses are fixed or separately accounted for; and the reciprocal chart stays on a nonzero branch.

There are two different uses of fixed data. Within one conservative boundary-value problem, a prescribed source has $\delta\mu = 0$. To measure the response to that source, one compares nearby problems with different assigned μ while controlling the remaining data. The response is a derivative across that family, just as a susceptibility compares solutions at neighboring applied fields.

A fixed physical clock prescription does not mean $\delta\kappa = 0$. The geometry and its connection can change while the convention defining the time generator remains fixed. Conversely, rescaling the physical clock is not automatically a harmless auxiliary-gauge change.

Not every freely chosen boundary function is guaranteed to admit a compatible solution of the gravitational constraints. The manuscript's variational identity is conditional on the admitted source space; it is not a theorem of existence and uniqueness for every proposed boundary problem. [Original full paper §II.A–B; Appendix C]

3.8 Hamilton–Jacobi meaning and units

In ordinary mechanics the action evaluated on a classical solution is a function of boundary data. Differentiating with respect to an endpoint coordinate yields its momentum; differentiating with respect to a time endpoint yields energy with the appropriate sign. Hamilton–Jacobi theory carries this idea to an action as a function, or functional, of boundary sources.

For the selected mixed gravitational action, its local source response reads

$$\delta I_{\rm os}\big|_{\text{other sources}} = -\int d\tau \, F_L(\tau) \delta L_s(\tau), \qquad F_L(\tau) = -\frac{\delta I_{\rm os}}{\delta L_s(\tau)}.$$

The derivative is functional because $L_s(\tau)$ is a function of time, not merely one number. The integration measure is part of the definition: action divided by time and length has force units. For a time-independent source over a prescribed interval,

$$-\frac{\partial I_{\rm os}}{\partial L_s} = \Delta\tau F_L.$$

This establishes the canonical length-source response without identifying U_∂ with the entire Hamiltonian. Its sign follows the stated boundary orientation and selected rate action. A material-force measurement or a protocol for extracting work would require additional physical structure. [Current compact paper §3; full manuscript §II and Appendix I]

4 The stationary corner interpretation

4.1 Boost angle and its momentum

The two directions normal to a spacelike cut form a Lorentzian plane. Two choices of normal frame can differ by a Lorentz boost. Its rapidity η is the hyperbolic-angle analogue of an ordinary rotation angle; for two suitable timelike observers in one local plane, relative speed is $v = c\tanh\eta$. At a boundary joint the relevant quantity is the relative boost of the boundary normals. It need not equal a cosmological screen's elementary velocity rapidity.

Einstein's corner structure contains, in the adopted orientation, the one-form

$$\Theta_{\text{corner}} = J_\eta \delta\eta, \qquad J_\eta = \frac{Ac^3}{8\pi G}.$$

This is the same sense of conjugacy as angular momentum times angle. J_η is action-valued and is a gravitational boost charge, not electric charge, rest mass, or ordinary black-hole spin. Its value changes when the area changes; calling it a charge does not imply conservation under every physical process. [CT95; BW12]

4.2 The length momentum explicitly

For a stationary boost flow over a fixed interval,

$$\eta = \frac{\kappa\Delta\tau}{c} = \frac{c\Delta\tau}{L_s}.$$

Consequently

$$p_L \equiv J_\eta \frac{\partial\eta}{\partial L_s} = -\frac{c\Delta\tau J_\eta}{L_s^2} = -\Delta\tau\frac{A\kappa^2}{8\pi G} = -\Delta\tau F_L,$$

and $J_\eta\delta\eta = p_L\delta L_s$.

p_L has ordinary momentum units (force times time); F_L is the response per unit boundary time. They are not the same object and neither is the pressure P_L. The symbols p_L, P_L, p_m, and p_B name different quantities. The stationary interval is essential to this simple integrated expression. For nonstationary boost data the relation is an integral with additional time/geometry variations as required; one cannot insert a changing cosmological rate into this stationary corner identity and call the result a complete moving-screen derivation.

4.3 Why $U_\partial L_s = cJ_\eta$ is revealing

The charge–length relation follows directly from $U_\partial = \omega_s J_\eta$ and $L_s = c/\omega_s$:

$$\boxed{\frac{U_\partial L_s}{c} = J_\eta = \frac{Ac^3}{8\pi G}.}$$

It says that energy multiplied by its reciprocal-rate time L_s/c is the normal boost charge. At fixed area, changing the clock-normalized rate changes U_∂ and L_s inversely while their product retains the area-normalized action. Under a constant physical-clock rescaling, that product is unchanged. This does not make F_L or P_L separately clock invariant.

The full paper already states $S/k_B = 2\pi U_\partial L_\kappa/(\hbar c)$ on the positive branch (Part I, Eq. S26; main Eq. 84), and both papers state $J_\eta = Ac^3/(8\pi G)$. Their equivalence was not foregrounded as this compact purely mechanical equality. The boost-Hamiltonian interpretation is established literature; the concise acceleration-length packaging is useful exposition of the manuscript's framework, not an independently new entropy law.

The relation uses signed U_∂, L_s, or positive magnitudes $|U_\partial|, L_\kappa$. At exactly $\kappa = 0$, the reciprocal coordinate is undefined; a finite branchwise limit is not permission to multiply zero by infinity at the endpoint.

4.4 Only afterward: the thermal dictionary

Where a regular Hawking–Unruh/KMS interpretation applies,

$$k_B T = \frac{\hbar \Omega_\kappa}{2\pi}, \qquad \frac{S}{k_B} = \frac{2\pi J_\eta}{\hbar}.$$

For positive κ, this gives

$$\omega_s \, dJ_\eta = \Pi_\partial \, dA = T \, dS, \qquad J_\eta \, d\omega_s = -F_L \, dL_\kappa = S \, dT.$$

The word "thermal" belongs here, not to the preceding charge–rate identity. The factor 2π is associated with the regular Euclidean angular period; the gravitational normalization supplies J_η. The quarter-area Einstein entropy follows with its conventional zero additive constant. No planar disk, microscopic tiling, or new thermal postulate has been derived by canceling the fours in $A = 4\pi R^2$.

5 Which horizons are actually covered?

Setting	Canonical support	Meaning of acceleration length
Rindler, stationary finite patch	Yes, in the stated clock/polarization	Exact observer–horizon proper distance c^2/a
Schwarzschild, stationary horizon	Yes, in the stated clock/polarization	$2R_H$ with time normalized at infinity; not generic proper radial distance
Other stationary Einstein Killing horizons	Same local scalar structure under its assumptions; other charges may be needed	Reciprocal of the selected surface gravity
Exact de Sitter horizon	Stationary null realization available, with consistent clock/orientation	R_A in the conventional static/CK magnitude
Generic FRW apparent-horizon history	The CK state identities hold, but not by direct application of the nonexpanding-null theorem	R_A by the independently specified CK calibration
Spherical dynamical Hayward horizon	Established Kodama–Noether potential and unified balance; not an unrestricted replacement for the null source theorem	Reciprocal cross-focusing rate, generally not R_A

The direct action result covers the specified stationary or nonexpanding null sector. Generic evolving FRW apparent-horizon histories have a separate screen-state construction. The algebraic boundary potential supports all three after their stated dictionaries are supplied. The exact FRW application is not thereby invalidated or merely approximate: what differs is its canonical provenance and its thermal-equilibrium status. [Original full paper §VI.C; Cai–Kim]

5.1 A marginal cut is not a nonexpanding null worldtube

At an apparent-horizon cut, one of the two null expansions vanishes. To find the next apparent-horizon cut, however, one follows the locus of marginal spheres, not necessarily that same family of null generators. The worldtube formed by those spheres can be timelike, spacelike, or null. Its area can change even though every constituent sphere is marginal in one null direction.

For flat dust FRW, $a(t) \propto t^{2/3}$ gives $R_A = 3ct/2$. Relative to the comoving flow, the horizon has radial speed $\dot{R}_A - HR_A = c/2$. Its worldtube is timelike and its area increases. The null, nonexpanding hypotheses therefore fail. The velocity rapidity is constant, although the Hayward rate is nonzero. This is why a simple worldtube rapidity derivative does not supply the required horizon boost rate. [Original full paper §VI.G; Faraoni]

There are special null cases, but nullness alone is not enough: the expansion must vanish along the actual generator of the tube. Exact de Sitter supplies the stationary, nonexpanding example. A radiation apparent horizon in flat FRW can be null while the relevant tube expansion is not zero.

5.2 What already holds in FRW

With arbitrary spatial curvature,

$$R_A = \frac{c}{\sqrt{H^2 + kc^2/a^2}}, \qquad \kappa_{\mathrm{CK}} = \frac{c^2}{R_A}, \qquad L_{\mathrm{CK}} = R_A.$$

Using the Einstein–FRW matter relation $E_{\mathrm{MS}} = \rho V_A = c^4 R_A/(2G)$ and areal volume $V_A = 4\pi R_A^3/3$,

$$P_{\mathrm{scr}} = \frac{\rho}{3} = P_L^{\mathrm{CK}}, \qquad E_{\mathrm{MS}} = U_\partial = F_L L_{\mathrm{CK}} = 3P_L^{\mathrm{CK}} V_A.$$

Here the acceleration-length displacement is exactly the areal-radius displacement, so $F_R = F_L = M\kappa$ and $P_R = P_L = P_{\mathrm{scr}}$. The same work is

$$F_L\, dL_{\mathrm{CK}} = F_R\, dR_A = P_R\, dV_A.$$

The complete mechanical state differential is

$$dE_{\mathrm{MS}} = \Pi_{\mathrm{CK}}\, dA_A - F_R\, dR_A = \Pi_{\mathrm{CK}}\, dA_A - P_R\, dV_A.$$

The thermal version uses $\Pi_{\mathrm{CK}} dA_A = T_{\mathrm{CK}} dS_A$. This is a constrained screen-state law, not the matter energy-flux law across a momentarily fixed horizon. The bulk-energy identification is explicit gravitational input, not a second density invented for presentation. No new bulk Friedmann dynamics are claimed.

6 Can the canonical argument be extended?

6.1 What can already be said on an expanding null boundary

There is a precise partial extension available from the same scalar one-form. Define $\mu_{\mathrm{acc}} = c^2\mu = \kappa + c^2\theta/2$. With the same fixed clock-measure and polarization conventions, its scalar part is

$$\Theta_\mu^{(0)} = \int d\tau \int_S \epsilon_S \left(\frac{c^2}{8\pi G}\delta\kappa + \frac{c^4}{16\pi G}\delta\theta \right).$$

Using $L_s = c^2/\kappa$ gives the exact scalar coordinate identity

$$\boxed{\Theta_\mu^{(0)} = -\int d\tau \int_S \epsilon_S P_L \delta L_s + \int d\tau \int_S \epsilon_S \frac{c^4}{16\pi G}\delta\theta.}$$

Therefore the familiar surface-gravity response survives as a **partial scalar coefficient**; an independent expansion response remains. Setting $\delta\theta = 0$ removes that displayed term, even at a nonzero fixed expansion, provided the remaining boundary conditions really admit the chosen variations. This last proviso is a geometric/source-space question, not answered by algebra alone.

Alternatively, on $\mu \neq 0$ one can use $L_\mu = 1/\mu$ and write the whole scalar sector as $-\int \epsilon_S P_\mu \delta L_\mu \, d\tau$ with $P_\mu = c^4/(8\pi G L_\mu^2)$. But L_μ is generally **not** c^2/κ. Neither version removes shear, generator, matter, clock, or corner responses. This is an algebraic consequence of the established scalar potential, added explicitly in this explanatory revision; it is not promoted to a complete dynamical horizon theorem.

6.2 What a full moving-boundary theorem would have to establish

"Generic moving null-boundary theorem" is not a particular standard theorem name. Two problems should be distinguished: an evolving but still null hypersurface with expansion/shear, and an apparent-horizon worldtube whose causal type may itself be non-null. The latter requires an appropriate timelike/spacelike or signature-aware treatment, not merely a null formula with time-dependent symbols.

A complete theorem for a selected horizon prescription would specify its geometry, embedding, physical evolution, matter boundary data, polarization, counterterms, and corner conditions. It would calculate the full on-shell boundary variation, then demonstrate which response is paired with the original intended $L_{\rm CK}$ or L_H, with all remaining source and transport terms retained. It must also distinguish a local response coefficient from an integrable Hamiltonian charge.

The papers already establish several pieces, including the general null scalar structure, the stationary boost check, spherical Noether identities, and the explicit timelike calculation. What is not established is their universal identification with the CK or Hayward rate on arbitrary moving FRW screens. The timelike counterexample below disproves that identification for **that ensemble**, not for all ensembles. The stretched-Carrollian formalism is relevant because it retains clock, stretching, and other responses in a common description. [Full paper Appendix C; Freidel–Jai-akson §5.1]

7 Why ζ_B is defined that way

Use $c = 1$, as in Appendix C. A spherical timelike worldtube has induced metric $-N_B^2 dt^2 + R_B^2 d\Omega_2^2$. The lapse N_B relates proper boundary time to the coordinate time. The on-shell Dirichlet boundary response is

$$\delta I_D\big|_B = \int dt \, [-E_B \delta N_B + N_B p_B \delta A_B] \, .$$

E_B is the unreferenced Brown–York energy conjugate to the clock variable; p_B is the isotropic two-dimensional surface stress, not bulk pressure. To prescribe stress rather than area, choose

$$I_M = I_D - \int dt \, N_B A_B p_B .$$

The result is

$$\delta I_M\big|_B = \int dt \, [-E_B \delta N_B - A_B \delta(N_B p_B)] \, .$$

Only now introduce the notation

$$\zeta_B \equiv -8\pi G N_B p_B,$$

which puts the second term into $A_B\delta\zeta_B/(8\pi G)$. The factor $8\pi G$ removes the conventional Einstein normalization; the lapse places the local surface stress in the selected clock convention; the minus sign matches the chosen orientation/polarization. **It is a normalization choice, not a derivation that this source equals surface gravity.**

The geometry gives its independent content. With outward normal n^a, boundary unit timelike vector u^a, and the paper's extrinsic-curvature convention,

$$a_n = n_a u^b \nabla_b u^a, \qquad b_B = \frac{n^a \nabla_a R_B}{R_B}, \qquad p_B = \frac{a_n + b_B}{8\pi G},$$

so

$$\boxed{\zeta_B = -N_B(a_n + b_B).}$$

a_n is the normal component of the observers' acceleration; b_B measures the normal change of the transverse radius per radius. In spherical symmetry it is the angular extrinsic-curvature eigenvalue. The broader quantity is therefore a **clock-weighted combination of normal acceleration and transverse extrinsic curvature**, namely the surface-stress source of this boundary problem.

On $\zeta_B \neq 0$, the reciprocal coordinate $L_B = 1/\zeta_B$ gives

$$\delta I_M\big|_B = \int dt[-E_B\delta N_B - F_B\delta L_B], \qquad F_B = \frac{A_B\zeta_B^2}{8\pi G}.$$

This variational calculation is meaningful within the specified action and matter/endpoint conditions. Its square-law shape also reflects using a reciprocal coordinate. An auxiliary $U_B = A_B\zeta_B/(8\pi G)$ is not generally E_B or the Misner–Sharp energy. The action's force-like response becomes the paper's original horizon response only after $\zeta_B = \kappa/c^2$ is independently established with matching normalization.

In flat dust FRW the appendix finds $\zeta_B = -2/R_A$, whereas $\kappa_{\rm CK} = 1/R_A$ and $\kappa_H^s = -1/(4R_A)$ in these units. The mismatch is real and cannot be removed merely by flipping orientation. In stationary stretched Schwarzschild and de Sitter limits, a matching horizon rate is recovered. Thus this appendix is best read as a **scope test of the larger extrinsic boundary response**, not as another proof of the CK formula. [Original full paper Appendix C]

8 How the pressure enters membrane mechanics

The surface stress Π_∂ has units of energy per area or force per length. The pressure P_L has force-per-area units. They are linked by

$$\Pi_\partial(L_s) = \frac{c^4}{8\pi G L_s} = P_L L_s, \qquad d\Pi_\partial = -P_L dL_s.$$

On a nonuniform cut, let $L_s = L_s(x)$. The tangential derivative D_i is the covariant spatial gradient using the cut metric. Pulling back the source-space identity means evaluating it on this spatially varying field:

$$D_i\Pi_\partial = -P_L D_i L_s, \qquad -D_i\Pi_\partial = P_L D_i L_s.$$

The last expression is the familiar surface-pressure-gradient contribution to tangential membrane momentum balance, expressed through the acceleration-length gradient. There is no new force law hidden in the word

"pullback"; it is the geometric chain rule. The physical content is the connection to the already established membrane stress.

Normal balance is a different projection. In the matched static timelike-screen convention of the paper,

$$0 = \Delta p + \gamma \mathcal{H} = \Delta p - P_L L_s \mathcal{H},$$

where $\gamma = -\Pi_\partial$ is oriented surface tension, $\mathcal{H}$ is the trace of the cut's spatial extrinsic curvature (the sum of principal curvatures), and Δp is the appropriate oriented normal-pressure difference including the screen equation's curvature contribution. $\mathcal{H}$ is not the cosmological Hubble rate H. On a round sphere in Euclidean three-space this trace is $2/R$.

Thus the corresponding traction is $\Delta p = P_L L_s \mathcal{H}$, not universally P_L by itself. The dimensionless factor $L_s \mathcal{H}$ converts the normal-scale response to curvature pressure. The full dynamical screen equation also contains expansion, shear, normal-connection, and lapse-gradient contributions. Those do not disappear by substituting $\Pi_\partial = P_L L_s$. A screen normal rate must be matched to the desired horizon prescription rather than identified by its symbol alone. [Original full paper §IV.C–D; Freidel–Yokokura]

9 The selected clock and boundary energy

The boundary potential uses the same surface-gravity normalization and acceleration length throughout its mechanical and canonical descriptions. A change of time normalization changes the rate and the energy assigned to that evolution; keeping the clock prescription fixed does not mean holding the surface gravity fixed between different geometries.

For a stationary horizon, the area–boost coefficient and rate are

$$J_\eta = \frac{Ac^3}{8\pi G}, \qquad \omega = \frac{\kappa}{c} = \frac{c}{L_s}.$$

Thus $U_\partial = J_\eta \omega = J_\eta/(L_s/c)$. The selected corner one-form $J_\eta \delta\eta$ gives $-\Delta\tau F_L \delta L_s$ under a fixed-interval source variation. This is the framework's own macroscopic clock and length, not a change to another energy or distance prescription.

The oriented raw corner action, the selected canonical one-form and a full Hamiltonian are distinct. Their signs and boundary terms must be matched explicitly. In particular, a cosmological matter Killing-energy variation is not the Misner–Sharp energy change of the varying horizon sphere. The changing-screen balance does not remove the physical fact that positive added matter can reduce the cosmological horizon area. [CT95; BW12; IW94; 6 October 2026 seven-page revision §§3–5]

10 Why Hayward work is not CK work

The geometric prescriptions differ:

$$\kappa_H^s = -\frac{c^2}{R_A}\left(1 - \frac{\dot{R}_A}{2HR_A}\right), \qquad \kappa_{\mathrm{CK}} = \frac{c^2}{R_A}.$$

Hayward's rate probes normal cross-focusing and depends on evolution; CK supplies the instantaneous equilibrium radius scale. In Einstein–FRW, with $w = p_m/\rho$ and $\chi = \kappa_H^s/\kappa_{\mathrm{CK}} = (3w-1)/4$,

$$\widetilde{U}_\partial = \chi E_A, \qquad L_H^s = \frac{R_A}{\chi}, \qquad P_L^H = \chi^2 P_{\mathrm{scr}}.$$

The superscript H denotes evaluation of the independent-source response on this signed family, on $\kappa_H^s \neq 0$:

$$L_H^s = \frac{c^2}{\kappa_H^s}, \qquad F_L^H = -\left(\frac{\partial U_\partial(A, L_s)}{\partial L_s}\right)_A\Bigg|_{A=A_A,\, L_s=L_H^s}.$$

Differentiate at fixed area first, then substitute the Hayward data. Differentiating the family length in its radius and equation-of-state directions gives

$$\boxed{F_L^H\, dL_H^s = \chi P_{\rm scr}\, dV_A - E_A\, d\chi.}$$

The extra term is essential when the equation of state changes. Even at constant w, the coefficient differs from CK's unless the signed or magnitude prescriptions coincide appropriately. The original full paper already derives this exact relationship; it is not merely an unexplained warning that the two work terms differ.

Hayward's matter work density is $W = (\rho - p_m)/2$. The projected unified first law is $d_H E_A = T_H^s d_H S_A + W d_H V_A$, while the CK screen-state law is $dE_A = T_{\rm CK} dS_A - P_{\rm scr} dV_A$. Different area coefficients compensate the different work allocations on the physical horizon trajectory. They are compatible balances, not identical terms. In radiation FRW the Hayward rate vanishes while CK pressure remains finite, making a universal identification impossible. Exact de Sitter is a special stationary agreement in magnitudes. [Original full paper §VI.E–F]

10.1 Signed radial pressure and source reconstruction

The positive screen coefficient gives the total energy-density scale. The signed Hayward radial coefficient adds the contrast associated with the evolving normal geometry:

$$P_R^H = \frac{M_A \kappa_H^s}{A_A} = \frac{c^2 \kappa_H^s}{8\pi G R_A} = \frac{p_m - P_{\rm scr}}{4}, \qquad M_A = E_A/c^2.$$

$$\rho = 3P_{\rm scr}, \qquad p_m = P_{\rm scr} + 4P_R^H, \qquad P_L^H = \frac{(P_R^H)^2}{P_{\rm scr}}.$$

The radial coefficient is signed, the length coefficient is nonnegative, and neither is Hayward's work density $W = (\rho - p_m)/2$. The mass is the horizon Misner–Sharp mass, not the mass equivalent of the signed boundary potential. The complete work remains

$$F_L^H dL_H^s = P_R^H dV_A - \frac{3}{4} E_A dw.$$

P_R^H is the volume coefficient at fixed w. The instantaneous reconstruction does not require fixed w; the full work along a changing-w history retains the second term. At $\kappa_H^s = 0$, use the regular rate/pressure variables; its reciprocal length is undefined.

A geometric history supplies the two coefficients without a prior matter equation of state:

$$P_{\rm scr} = \frac{c^4}{8\pi G R_A^2}, \qquad P_R^H = -P_{\rm scr}\left(1 - \frac{\dot{R}_A}{2HR_A}\right).$$

$$p_m = P_{\rm scr}\left(\frac{2\dot{R}_A}{HR_A} - 3\right).$$

Identifying these quantities with the bulk source uses Einstein's equations. This is an on-shell inverse reconstruction, not a new material equation of state or a derivation of matter microphysics. A matter model is still required to predict the geometry from initial data. The displayed ratio assumes $H \neq 0$ and finite R_A; use the regular normal rate at a zero-H instant. Flat expanding FRW has $HR_A = c$, so radius and radius rate suffice; curved FRW also uses H. Vacuum stress is included in total density and pressure.

The acceleration and conservation equations become

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{c^2}(P_{\rm scr} + 2P_R^H), \qquad \dot{P}_{\rm scr} + 4H(P_{\rm scr} + P_R^H) = 0.$$

They are the same Einstein–FRW equations, not an independent closure relation.

10.2 Causal history in the two pressure variables

For spatially flat expanding FRW, the horizon tangent norm in cosmic time is

$$-c^2 + (\dot{R}_A - HR_A)^2 = 4c^2 \frac{P_R^H(P_{\rm scr} + P_R^H)}{P_{\rm scr}^2}.$$

It is timelike for $-P_{\rm scr} < P_R^H < 0$, null at either endpoint, and spacelike outside. Persistent vacuum domination gives the de Sitter nonexpanding null history; dust has $P_R^H = -P_{\rm scr}/4$ and a timelike history; radiation has $P_R^H = 0$ and a null but area-growing history. A marginal sphere, a null history and a nonexpanding history are distinct conditions. This norm is not a local material speed, new evolution equation, or complete moving-boundary action theorem. Faraoni, Eqs. (74)–(75), Table 1.

10.3 Rates, clocks and levels of support

Schwarzschild's usual surface gravity is evaluated at its horizon and normalized by the stationary clock at infinity; it is not measured at infinity. A local clock redshifts the rate and potential, without changing the geometry into cosmological geometry. Geometry/evolution and clock normalization are separate issues.

- **Rindler:** observer-normalized normal separation and independent planar area; no enclosed spherical mass.
- **Schwarzschild:** stationary infinity-normalized rate, $L = 2R$ and $U = Mc^2/2$; radial pressure is $2P_L$.
- **Exact de Sitter:** a stationary overlap with the appropriate static clock and matched orientation. Neighboring de Sitter radii generally change the cosmological source; they are not automatically fixed-Λ variations.
- **Instantaneous equilibrium screen in evolving FRW:** the assigned positive rate gives $L = R_A$, $U = E_A$ and $P_R = P_L = P_{\rm scr}$. The screen-state identity is exact under its inputs, rather than a stationary-worldtube or universal quantum-thermality claim.
- **Full Hayward FRW:** signed normal rate, different potential and work allocation, and on-shell source reconstruction. Matching every source and embedding term in a complete moving-boundary action remains separate.

Read the two responsive realization/support tables.

10.4 Precise antecedents and the canonical source trail

Ong's 2023 treatment, §IV Eqs. (10)–(16), obtains thermodynamic $E = M/2 = TS$ using shell volume $4\pi R^2\epsilon$ and adopted $P = 1/(32\pi R\epsilon)$ at fixed thickness. It also publishes spherical $P = 1/(16\pi R^2)$, but declines it in the chosen Smarr construction. The shell pressure is not generally half the spherical one. Here $U = Mc^2/2$ is the specified boundary potential while ADM energy remains Mc^2; its length response requires

neither a shell thickness nor a maximum-force premise. Matching a formula is a limited antecedent, not equivalence of energy prescription or cross-horizon mechanics.

The principal canonical calculation is the selected null-source derivation. The stationary corner calculation gives a complementary interpretation and check using the same Einstein area normalization, physical clock and stationary boost relation. Their agreement is not an independent empirical test.

- Hopfmüller–Freidel 1611.03096v2, Eq. (1): the scalar area coefficient is $\hat{\kappa} + (D-3)\theta/(D-2)$; in four dimensions $\mu = \hat{\kappa} + \theta/2$.
- Hopfmüller–Freidel 1802.06135v1, Eqs. (3)–(4): the mixed scalar and its source polarization. Both papers use $8\pi G = 1$; the null volume retains the boundary-time measure.
- Odak–Rignon-Bret–Speziale 2309.03854v3, Eqs. (3.26)–(3.28), (4.6), (4.8): scalar exchange and mixed conditions, with spin-two/generator restrictions and a joint area condition. Their $16\pi G = c = 1$ conventions give the same scalar factor only after matching generator, normalization, orientation and measure.

These correspondences identify the canonical antecedents and their conventions. The manuscript's Appendix I supplies the direct local Einstein–Hilbert current, selected subtraction, endpoint identity and length pull-back under explicit restrictions. Its scope remains that local boundary problem; arbitrary moving boundaries, matter responses and global corner completions require their own treatment. The stationary Wald-channel lift likewise remains separate from a complete higher-curvature theorem: the cited Lovelock surface-term variations have order-dependent normalization and restricted variations, while membrane pressure has an additional theory/dimension factor.

11 Reading the full paper: results and appendices

The main paper contains the new force–pressure identification, canonical conditions, and geometric realizations. The exact Schwarzschild products and inverse-energy/entropy relations remain there. Optional physical interpretations are not premises of that argument.

Supporting material	Role in the paper
Appendix D	Exact compliance, modulus, and Hessian calculations; not a material stability theorem.
Appendix E	Additional cosmological identities and operational tests; the qualifications remain with the comparisons.
Appendix F	Inflationary pressure/entropy formulas using an imported quantum spectrum and its extra assumptions.
Appendix G	Proposed kinetic/potential and virial reading, Newtonian stress correspondence, and conditional quantum-phase interpretation. No separate derivation of the proposed physical readings is claimed.
Appendix H	Conditional Planck-scale benchmarks; the scale alignments are imposed choices, not quantum-gravity predictions. The $G/2$ comparison is not a proposed modification.
Appendix I — current edition	Direct local Einstein–Hilbert current, explicit rate-action prescription, endpoint and clock conversion, and counterexamples defining the restricted length response.

An appendix separates these discussions from the principal result; it does not make a hypothesis established. The effective gas-particle count, constructed spherical Rindler family, and quantum-atmosphere radius comparisons are no longer part of the current presentation.

12 What the revised presentations do

The 29 October presentation states the canonical force directly as the coefficient of an allowed length-source variation in the selected action. The one-page summary, compact paper and full manuscript distinguish that response from an assertion that the boundary potential is the entire Hamiltonian. The compact paper identifies the local subtraction, clock map and variation class; the Hayward definitions explicitly preserve differentiation before family restriction. The current PDFs and their editable sources now describe the same edition.

The 22 October consolidated revision adds signed Hayward radial-pressure reconstruction and causal history, separates geometry and support in the realization dictionary, and records the precise Ong and null-source comparisons. The proposed interpretations remain in their approved appendices.

The 20 October 2026 revision states the new force and pressure identification consistently, consolidates why acceleration length is geometrically preferred, and adds the Questions and objections reader. It preserves the established inputs, sector-specific support, and exact spherical work conversion.

The radial-work presentation makes Schwarzschild's $F_L\,dL = M\kappa\,dR = P_R\,dV$, $P_R = 2P_L$ and equal fractional changes prominent, while retaining $L = R_A$ in equilibrium FRW and the independent planar area in Rindler. The All About L reader adds a short continuous explanation of the acceleration scale. The Concept index and standalone glossaries use the same force and pressure distinctions.

The one-page presentation starts from area and acceleration length, writes the gravitationally specified potential, and introduces surface stress, force and pressure through one differential. Its mechanical identity comes before the thermal dictionary; the same page gives the horizon realizations, screen law, cosmological pressure equations and a compact canonical source statement.

The seven-page paper follows the same structure more technically. It gives the primary null-source proof, keeps the corner transformation as a stationary interpretation, identifies de Sitter as the stationary cosmological overlap, and separates the exact equilibrium-FRW calibration from a complete moving-boundary claim.

The full manuscript separates the main mechanical argument from supporting calculations in Appendices D–F and optional interpretations in G–H. Its opening, canonical bridge, clock explanation and cosmological scope now match the main Summary. The Path and Motivation readers retain their question-led narratives; updated supporting cards no longer splice the coupling-first route into those stories. All three current PDFs have editable sources. Historical citation PDFs remain unchanged.

13 Source map and reading links

The glossary pages use stable internal anchors and give source locations in the original manuscripts. Original section numbers are retained in the descriptions even when revised pagination changes. The package includes reference copies of the supplied seven- and 41-page PDFs, the revised LaTeX sources, and separate revision notes.

- **Current compact paper and full manuscript:** *Horizon Boundary Mechanics*, 29 October 2026. Compact §3 identifies the selected action response; full §II and Appendix I give the detailed prescription and local calculation. Compact PDF · Full PDF. Existing older page-specific links below continue to identify their cited editions.
- **Original seven-page manuscript:** *Horizon Pressure from the Einstein Boundary Potential*, supplied PDF. §§2–3: mechanical and canonical derivation; §3.1: boost check; §4: mass-law distinction; §5: horizon realizations; §6: thermality; §7: scope. Reference PDF.
- **Original full manuscript:** *Surface Gravity Squared as Gravitational Boundary Pressure*, supplied 41-page PDF. §§II–III: source and clock; §IV: membrane and thermal structure; §V: stationary

sectors/FGP; §VI: cosmology; §VII: integrability/dynamics; §VIII: Wald extension; Appendix C: timelike comparison. Reference PDF.

- **HF17:** Hopfmüller–Freidel, *Gravity degrees of freedom on a null surface.* arXiv:1611.03096.
- **HF18:** Hopfmüller–Freidel, *Null conservation laws for gravity.* arXiv:1802.06135.
- **ORS23:** Odak–Rignon-Bret–Speziale, *General gravitational charges on null hypersurfaces.* arXiv:2309.03854.
- **CT95:** Carlip–Teitelboim, *The off-shell black hole.* arXiv:gr-qc/9312002.
- **BW12:** Bianchi–Wieland, *Horizon energy as the boost boundary term in general relativity and loop gravity.* arXiv:1205.5325.
- **IW94:** Iyer–Wald, *Some properties of Noether charge and a proposal for dynamical black hole entropy.* arXiv:gr-qc/9403028.
- **Harlow–Wu:** *Covariant phase space with boundaries.* arXiv:1906.08616.
- **FGP:** Frodden–Ghosh–Perez, *Quasilocal first law for black hole thermodynamics.* arXiv:1110.4055.
- **Cai–Kim:** *First law of thermodynamics and Friedmann equations of FRW universe.* arXiv:hep-th/0501055.
- **Faraoni:** *Cosmological apparent and trapping horizons.* arXiv:1106.4427.
- **Freidel–Yokokura:** *Non-equilibrium thermodynamics of gravitational screens.* arXiv:1405.4881.
- **Freidel–Jai-akson:** *Geometry of Carrollian stretched horizons*, §5.1. arXiv:2406.06709.