

Horizon boundary mechanics

Area, acceleration length, and horizon work

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Identifying acceleration length $L = c^2/\kappa$ as a geometrically normalized work coordinate yields a new generalized boundary force and normal-scale pressure from the established Einstein boundary potential.

Geometry and the mechanical responses

For a specified clock and positive surface gravity κ , use the horizon area A and

$$L = \frac{c^2}{\kappa}, \quad U(A, L) = \frac{A\kappa c^2}{8\pi G} = \frac{c^4}{8\pi G} \frac{A}{L}. \quad (1)$$

The specified Einstein boundary prescription fixes this potential [1]. Its complete differential is

$$dU = \frac{c^4}{8\pi GL} dA - \frac{Ac^4}{8\pi GL^2} dL = \Pi dA - F_L dL. \quad (2)$$

For the mass equivalent $M_U = U/c^2$, the responses and force density are

$$\Pi = \left(\frac{\partial U}{\partial A} \right)_L = \frac{U}{A}, \quad F_L = - \left(\frac{\partial U}{\partial L} \right)_A = \frac{U}{L} = M_U \kappa, \quad (3)$$

$$P_L = \frac{F_L}{A} = \frac{\Pi}{L} = \frac{\kappa^2}{8\pi G}, \quad U = \Pi A = F_L L = P_L A L. \quad (4)$$

Surface stress pairs with area change; force pairs with horizon-scale change: $A d\Pi = -F_L dL$. Rindler fixes the length normalization. Other coordinates preserve work when their coefficients transform.

Thermal translation and stationary horizons

The Hawking–Unruh temperature and Einstein area entropy give

$$T = \frac{\hbar \kappa}{2\pi k_B L}, \quad S = \frac{k_B A}{4\ell_P^2}, \quad \ell_P^2 = \frac{\hbar G}{c^3}, \quad U = TS, \quad \Pi dA = T dS. \quad (5)$$

Rindler: $L = c^2/a_{\text{obs}}$ is the normal horizon distance on the instantaneous rest slice; planar patch area A is independent. *Schwarzschild:* $A = 4\pi R^2$, $L = 2R$, $U = Mc^2/2$, with time normalized at infinity. For areal volume $V = 4\pi R^3/3$,

$$F_L dL = F_R dR = M\kappa dR = P_R dV, \quad P_R = 2P_L = \frac{\kappa^2}{4\pi G}. \quad (6)$$

Since $dL/L = dR/R$, the work describes the same horizon-size change. Area and volume co-vary; $U = 3P_R V$. This reveals the mass-energy decomposition $Mc^2 = TS + F_L L$, with both products equal to U , while retaining $d(Mc^2) = T dS$.

Cosmological payoff

For the instantaneous equilibrium FRW screen, $L = R_A$, so $P \equiv P_R = P_L$. With $V = 4\pi R_A^3/3$, the Einstein–Misner–Sharp identification gives [2]

$$U = E = \frac{c^4 R_A}{2G} = \rho V, \quad P = \frac{\rho}{3}, \quad \boxed{dE = T dS - P dV}. \quad (7)$$

For flat FRW, $R_A = c/H$ and the expansion, acceleration, and conservation laws are

$$\boxed{H^2 = \frac{8\pi G}{c^2} P}, \quad \boxed{\frac{2\ddot{a}}{ac^2} = -\kappa_E (P + p_m)}, \quad \boxed{\dot{P} + H(3P + p_m) = 0}, \quad (8)$$

Here $H = \dot{a}/a$, $\kappa_E = 8\pi G/c^4$, and p_m is total bulk pressure, distinct from P ; vacuum contributions are included consistently. For curved FRW, $R_A = c/\sqrt{H^2 + kc^2/a^2}$ and $1/R_A^2 = \kappa_E P$. These are the Einstein–FRW equations in screen-pressure variables.

Canonical basis and scope

With fixed clock, cuts, and the stated mixed boundary prescription, the scalar nonexpanding-null response ($\theta = \delta\theta = 0$) is [1]

$$\frac{\Theta_\kappa^{(0)}}{\Delta\tau} = \frac{Ac^2}{8\pi G} \delta\kappa = -\frac{Ac^4}{8\pi GL^2} \delta L = -F_L \delta L. \quad (9)$$

The new force and pressure are identified in this selected action response, not introduced as a new interaction. Stationary Rindler, Schwarzschild and de Sitter realize this sector. Evolving FRW uses a separate instantaneous equilibrium-screen assignment, not a stationary worldtube. Thermal input is unnecessary for F_L and P_L .

[1] Hopfmüller–Freidel, 1611.03096; Odak et al., 2309.03854; Parattu et al., 1303.1535.

[2] Cai–Kim, hep-th/0501055; Akbar–Cai, hep-th/0609128. Details and scope: the companion seven-page and full manuscripts.