Horizon pressure and Friedmann cosmology

Acceleration length expresses the Einstein boundary's normal-rate response as force–displacement work. The same pressure organizes energy, curvature, acceleration and conservation in the Cai–Kim FRW realization.

1. From the canonical rate response to force and pressure

For uniform stationary data on a prescribed-clock, nonexpanding Einstein null boundary ($\theta = \delta\theta = 0$), the selected scalar rate-source polarization gives [1]

$$\frac{\Theta_\kappa^{(0)}}{\Delta\tau} = \frac{Ac^2}{8\pi G}\,\delta\kappa = -F_L\,\delta L_\kappa, \qquad L_\kappa = \frac{c^2}{\kappa}, \qquad \kappa > 0. \tag{1}$$

Here L_κ is the light-travel length associated with timescale c/κ, not a universal proper distance. The established boost charge $J_\eta = Ac^3/(8\pi G)$ and normal rate $\Omega_\kappa = \kappa/c$ give [2]

$$\boxed{U_\partial = \Omega_\kappa J_\eta = \frac{Ac^2\kappa}{8\pi G}, \qquad U_\partial L_\kappa = cJ_\eta.}$$

Their differential is purely mechanical: $\Omega_\kappa\, dJ_\eta = \Pi_\partial\, dA$ and $J_\eta\, d\Omega_\kappa = -F_L\, dL_\kappa$, with $\Pi_\partial = U_\partial/A$. Hence

$$dU_\partial = \Pi_\partial\, dA - F_L\, dL_\kappa, \qquad F_L = -\left(\frac{\partial U_\partial}{\partial L_\kappa}\right)_A = \frac{U_\partial}{c^2}\kappa, \qquad \boxed{P_L \equiv \frac{F_L}{A} = \frac{\kappa^2}{8\pi G}}. \tag{2}$$

Rindler and stationary Schwarzschild directly realize this canonical sector. Thermal inputs subsequently give $U_\partial = TS = F_L L_\kappa$. Standard Schwarzschild Smarr reads $Mc^2 = TS + F_L L_\kappa$, each product equal to $Mc^2/2$.

2. FRW energy identity and the Friedmann constraint

For spatially flat, expanding FRW, with $H = \dot a/a > 0$, select the instantaneous Cai–Kim calibration [3]:

$$R_A = \frac{c}{H} = L_\kappa, \qquad \kappa = cH, \qquad A = 4\pi R_A^2, \qquad V = \frac{4\pi R_A^3}{3}. \tag{3}$$

Use the standard Einstein–FRW Misner–Sharp identification $E_{\mathrm{MS}} = \rho V$ [4], where ρ is total bulk energy density. The boundary potential gives

$$E_{\mathrm{MS}} = \frac{c^4 R_A}{2G} = \rho V = U_\partial = TS = F_L L_\kappa = 3P_L V. \tag{4}$$

Dividing by V relates the boundary pressure to the usual Friedmann constraint:

$$\boxed{\frac{c^2H^2}{8\pi G} = \frac{\kappa^2}{8\pi G} = P_L = \frac{\rho}{3}}, \qquad H^2 = \frac{8\pi G}{c^2}P_L = \frac{8\pi G}{3c^2}\rho. \tag{5}$$

Since $dV = A\, dR_A = A\, dL_\kappa$, the mechanical differential becomes

$$dE_{\mathrm{MS}} = T\, dS - P_L\, dV, \qquad F_L = \frac{E_{\mathrm{MS}}}{c^2}\kappa = \frac{c^4}{2G}. \tag{6}$$

This is a screen-state law, not a matter-flux law; P_L is distinct from the bulk fluid pressure.

3. Curvature, acceleration and conservation

With total bulk pressure p_m and Einstein coupling $\kappa_E = 8\pi G/c^4$,

$$\boxed{\frac{1}{R_A^2} = \kappa_E P_L}, \qquad \boxed{{}^{(2)}\mathcal{R}_\perp \equiv \frac{2\ddot a}{ac^2} = -\kappa_E(P_L + p_m)}. \tag{7}$$

Here $1/R_A^2$ is the sphere's Gaussian curvature and ${}^{(2)}\mathcal{R}_\perp$ is the normal time–radial Ricci scalar. Screen pressure sets the horizon-curvature scale; its sum with bulk pressure governs acceleration, which begins when $p_m < -P_L$. Bulk conservation becomes

$$\boxed{\dot P_L + H(\rho + p_m) = 0}, \qquad \rho = 3P_L. \tag{8}$$

The bulk enthalpy density $\rho + p_m$ controls the evolution of screen pressure.

Spatial curvature. The flat case is shown for clarity. For general FRW use $R_A = c/\sqrt{H^2 + kc^2/a^2}$ and $\kappa = c^2/R_A$. Then $P_L = \rho/3$ and Eqs. (4), (6)–(8) retain their form; $V = 4\pi R_A^3/3$ is areal volume, not generally proper spatial volume.

Inputs and scope. The null response fixes clock, cut, interval and other sources in the selected polarization. An evolving FRW screen is a separate calibrated realization, not an application of the nonexpanding-null theorem. Thermal identities use $T = \hbar\kappa/(2\pi c k_B)$ and $S = k_B c^3 A/(4G\hbar)$.

[1] Hopfmüller–Freidel, 1802.06135; Odak et al., 2309.03854. [2] Bianchi–Wieland, 1205.5325.

[3] Cai–Kim, hep-th/0501055. [4] Akbar–Cai, hep-th/0609128.

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